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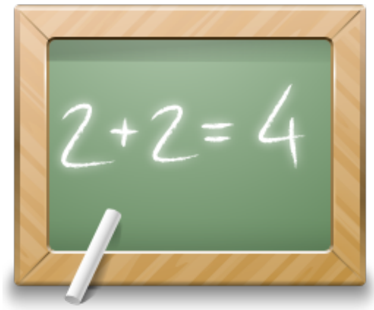
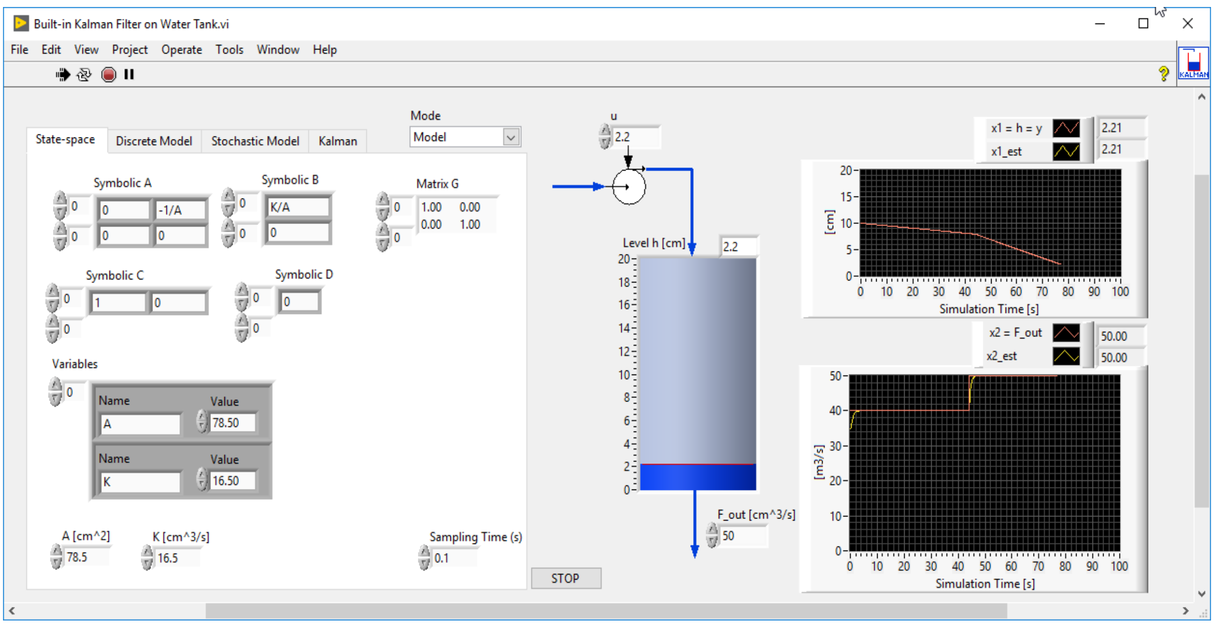


System Identification and Estimation in LabVIEW

Hans-Petter Halvorsen



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System Identification and Estimation in LabVIEW

Hans-Petter Halvorsen

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https://www.halvorsen.blog

Preface

This Tutorial will go through the basic principles of System identification and Estimation and how to implement these techniques in LabVIEW and LabVIEW MathScript.

LabVIEW is a graphical programming language created by National Instruments, while LabVIEW MathScript is an add-on to LabVIEW. LabVIEW MathScript has similar syntax as, e.g., MATLAB. LabVIEW MathScript may be used as a separate part (and can be considered as a miniature version of MATLAB) or be integrated into the graphical LabVIEW code using the MathScript Node.

The following methods will be discussed:

State Estimation:

- Kalman Filter
- Observers

Parameter Estimation:

- Least Square Method (LS)

System Identification

- Sub-space methods/Black-Box methods
- Polynomial Model Estimation: ARX/ARMAX model Estimation

Software

You need the following software in this Tutorial:

- LabVIEW
- LabVIEW Control Design and Simulation Module
- LabVIEW MathScript RT Module (LabVIEW MathScript)

"**LabVIEW Control Design and Simulation Module**" has functionality for creating Kalman Filters and Observers, but it also has functionality for System identification.

Table of Contents

Part I: Introduction

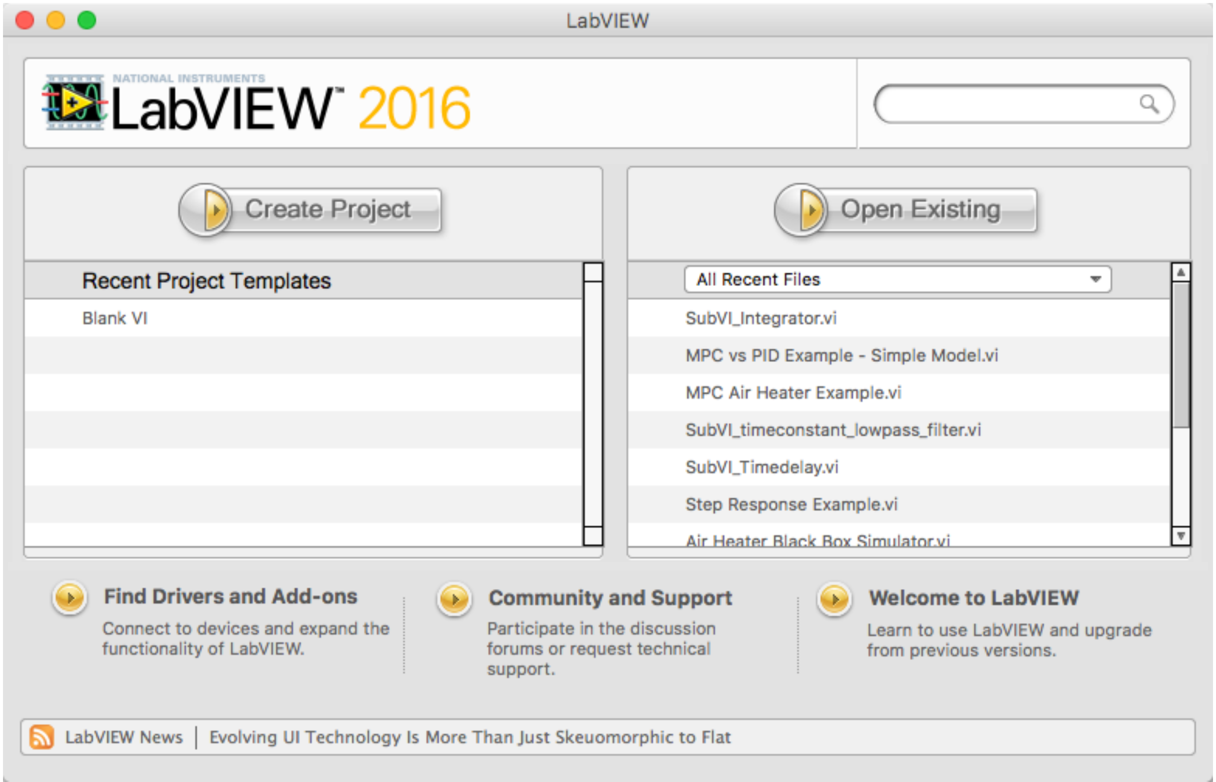
1 Introduction to LabVIEW and MathScript

In this Tutorial we will use LabVIEW and some of the add-on modules available for LabVIEW.

- LabVIEW
- LabVIEW MathScript RT Module
- LabVIEW Control Design and Simulation Module

1.1 LabVIEW

LabVIEW (short for **Lab**oratory **V**irtual **I**nstrumentation **E**ngineering **W**orkbench) is a platform and development environment for a visual programming language from National Instruments. The graphical language is named "G". LabVIEW is commonly used for data acquisition, instrument control, and industrial automation. The code files have the extension ".vi", which is an abbreviation for "Virtual Instrument". LabVIEW offers lots of additional Add-ons and Toolkits.



For more information about LabVIEW, please goto my Blog: https://www.halvorsen.blog and visit National Instruments at www.ni.com.

1.2 LabVIEW MathScript

MathScript is a high-level, text- based programming language. MathScript includes more than 800 built-in functions and the syntax is similar to MATLAB. You may also create custom-made m-file like you do in MATLAB.

MathScript is an add-on module to LabVIEW but you don't need to know LabVIEW programming in order to use MathScript.

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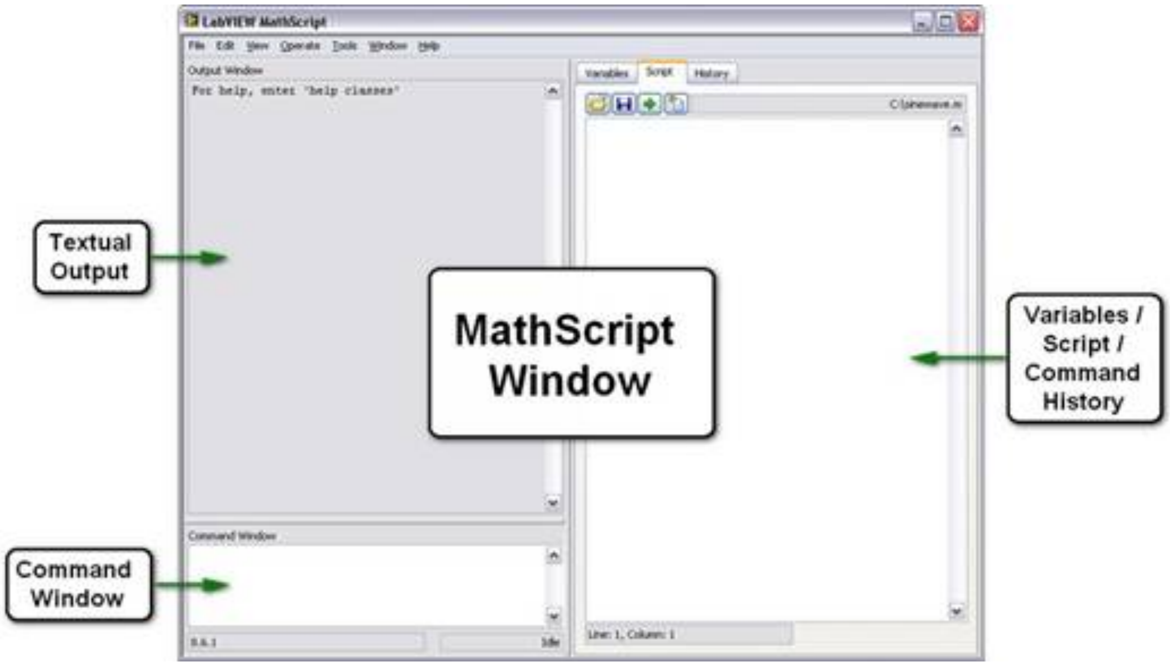
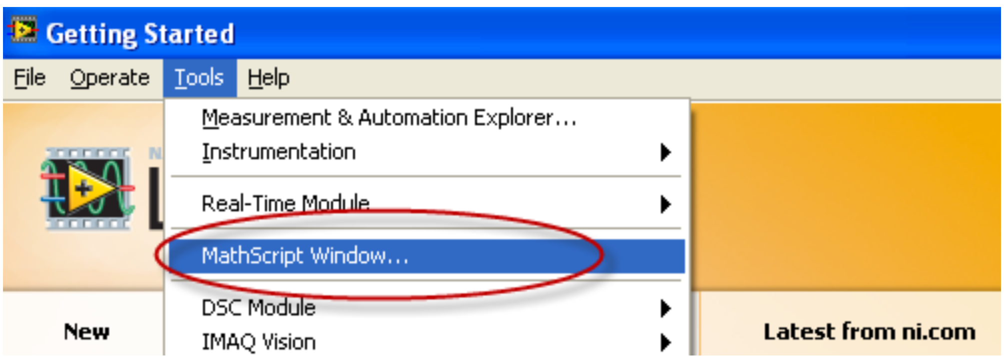


For more information about MathScript, please read the Tutorial "LabVIEW MathScript".

How do you start using MathScript?

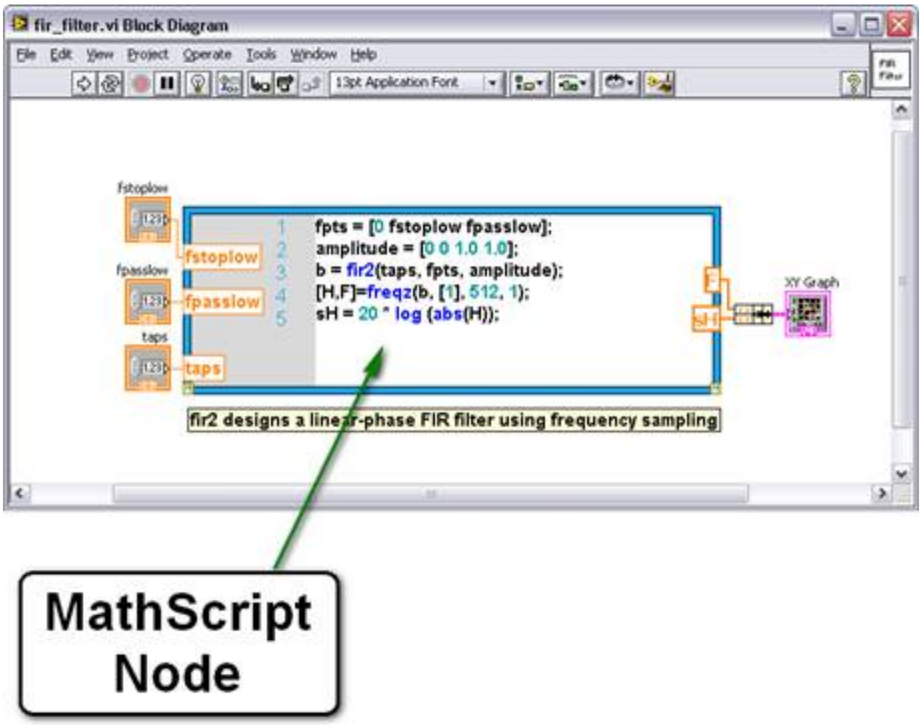
You need to install LabVIEW and the LabVIEW MathScript RT Module. When necessary software is installed, start MathScript by open LabVIEW. In the **Getting Started** window, select **Tools -> MathScript Window**...:



For more information about MathScript, please read the Tutorial “LabVIEW MathScript”.

MathScript Node:

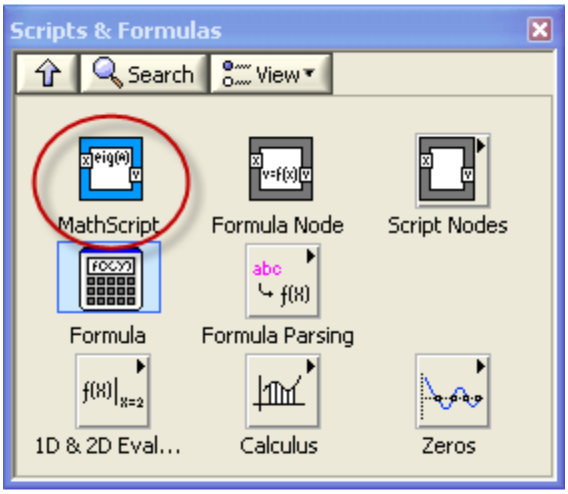
You may also use MathScript Code directly inside and combined with you graphical LabVIEW code, for this you use the “MathScript Node”. With the “MathScript Node” you can combine graphical and textual code within LabVIEW. The figure below shows the “MathScript Node” on the block diagram, represented by the blue rectangle. Using “MathScript Nodes”, you can enter .m file script text directly or import it from a text file.



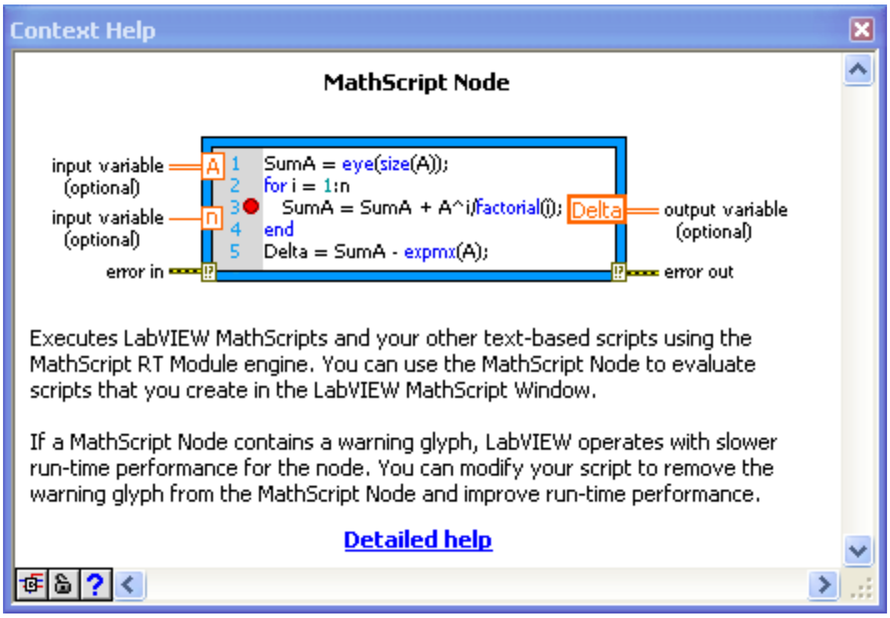
You can define named inputs and outputs on the MathScript Node border to specify the data to transfer between the graphical LabVIEW environment and the textual MathScript code.

You can associate .m file script variables with LabVIEW graphical programming, by wiring Node inputs and outputs. Then you can transfer data between .m file scripts with your graphical LabVIEW programming. The textual .m file scripts can now access features from traditional LabVIEW graphical programming.

The MathScript Node is available from LabVIEW from the Functions Palette: Mathematics → Scripts & Formulas



If you click Ctrl + H you get help about the MathScript Node:



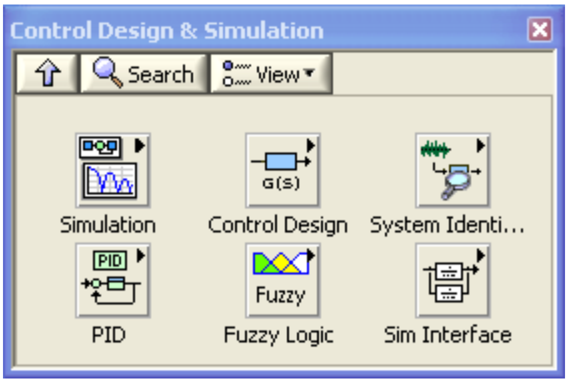
Click “Detailed help” in order to get more information about the MathScript Node.

2 LabVIEW Control and Simulation Module

LabVIEW has several additional modules and Toolkits for Control and Simulation purposes, e.g., "**LabVIEW Control Design and Simulation Module**", "**LabVIEW PID and Fuzzy Logic Toolkit**", "**LabVIEW System Identification Toolkit**" and "LabVIEW Simulation Interface Toolkit". LabVIEW MathScript is also useful for Control Design and Simulation.

- LabVIEW Control Design and Simulation Module
- LabVIEW PID and Fuzzy Logic Toolkit
- LabVIEW System Identification Toolkit
- LabVIEW Simulation Interface Toolkit

Below we see the Control Design & Simulation palette in LabVIEW:



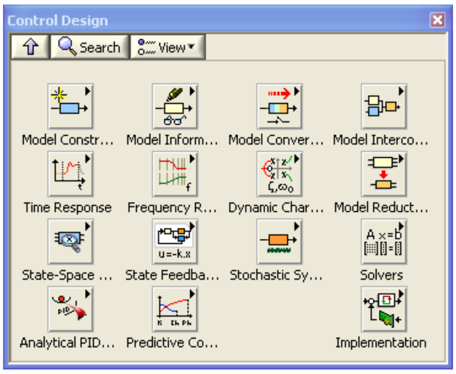
In this Tutorial we will focus on the VIs used for Parameter and State estimation and especially the use of Observers and Kalman Filter for State estimation.

If you want to learn more about Simulation, Simulation Loop, block diagrams and PID control, etc., I refer to the Tutorial "Control and Simulation in LabVIEW" This Tutorial is available from https://www.halvorsen.blog.

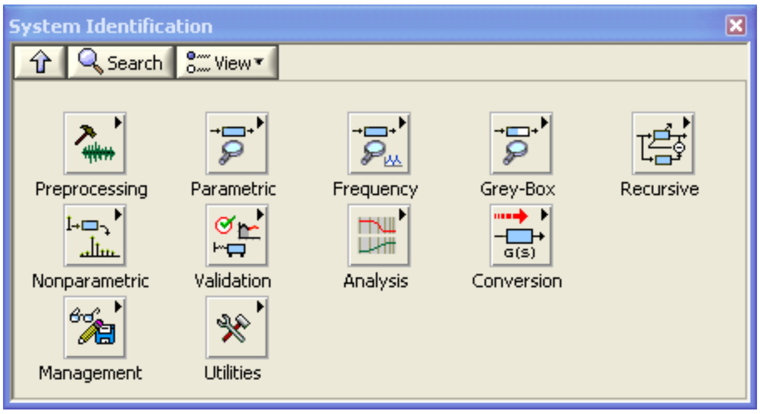
In this Tutorial, we will need the following sub palettes in the Control Design and Simulation palette:

- Control Design
- System Identification
- Simulation

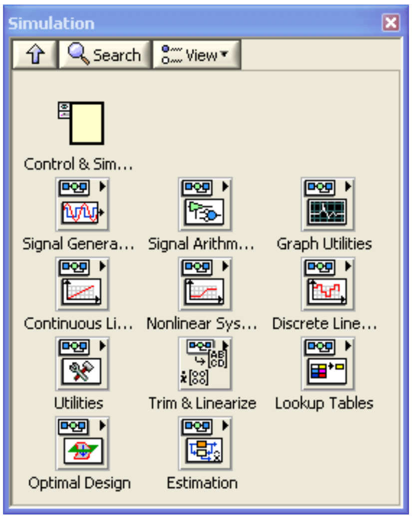
Below we see the Control Design palette:



Below we see the System Identification palette:



Below we see the Simulation palette:



In the next chapters we will go in detail and describe the different sub palettes in these palettes and explain the functions/Sub VIs we will need for System identification and Estimation.

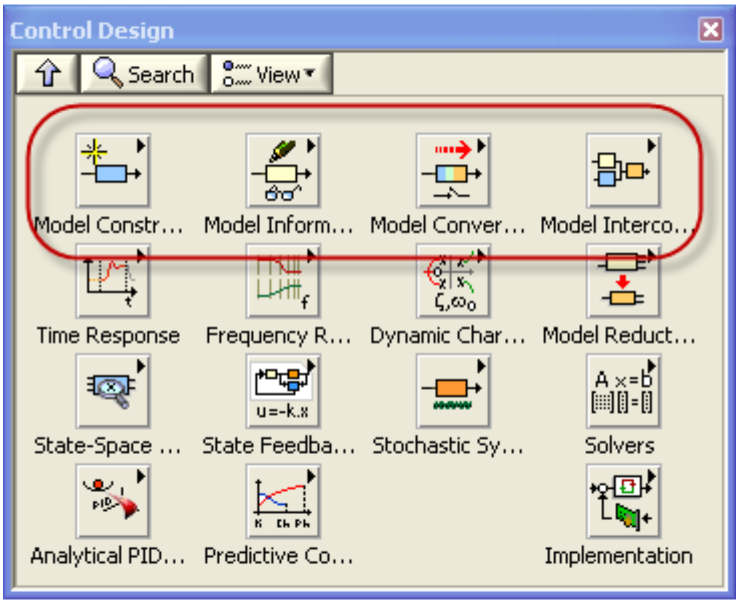
3 Model Creation in LabVIEW

When you have found the mathematical model for your system, the first step is to define/ or create your model in LabVIEW. Your model can be a Transfer function or a State-space model.

In LabVIEW and the "LabVIEW Control Design and Simulation Module" you can create different models, such as State-space models and transfer functions, etc.

In the Control Design palette, we have several sub palettes that deals with models, these are:

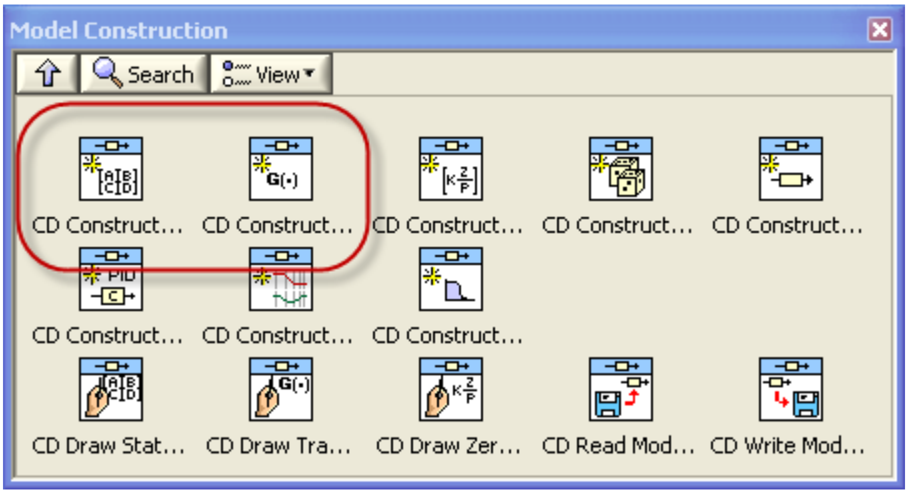
- Model Construction
- Model Information
- Model Conversion
- Model Interconnection



Below we go through the different subpalettes and the most used VIs in these palettes.

<u>"Model Construction" Subpalette:</u>

In this palette we have VIs for creating state-space models and transfer functions.



→ Use the Model Construction VIs to create linear system models and modify the properties of a system model. You also can use the Model Construction VIs to save a system model to a file, read a system model from a file, or obtain a visual representation of a model.

Some of the most used VIs would be:

 **CD Construct State-Space Model.vi**

 **CD Construct Transfer Function Model.vi**

These VIs and some others are explained below.

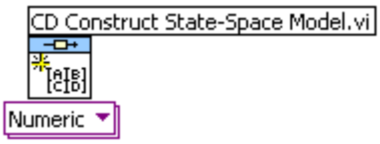
3.1 State-space Models

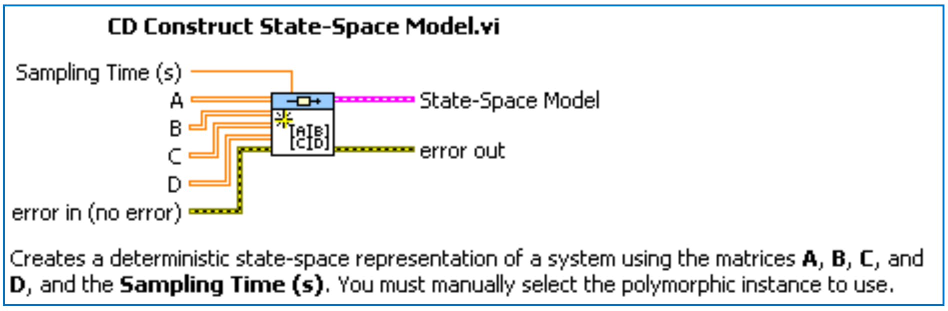
Given the following State-space model:

$$\dot{x} = Ax + Bu$$

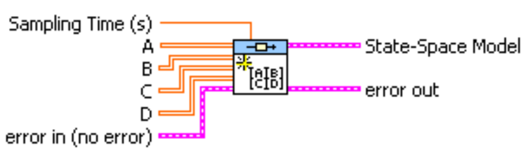
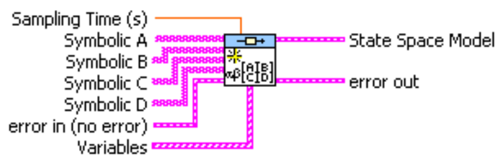
$$y = Cx + Du$$

In LabVIEW we use the "**CD Construct State-Space Model.vi**" to create a State-space model:



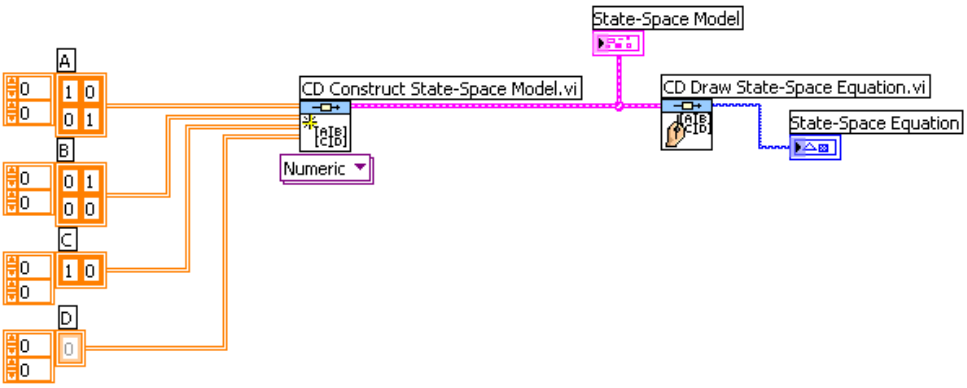


You may use numeric values in the matrices A,B,C and D or symbolic values by selecting ether “Numeric” or “Symbolic”:

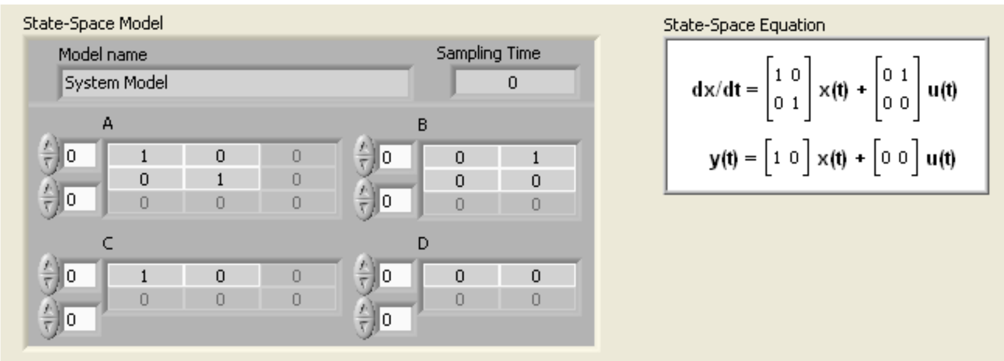
Numeric	Symbolic
	

Example: Create State-Space model

Block Diagram:



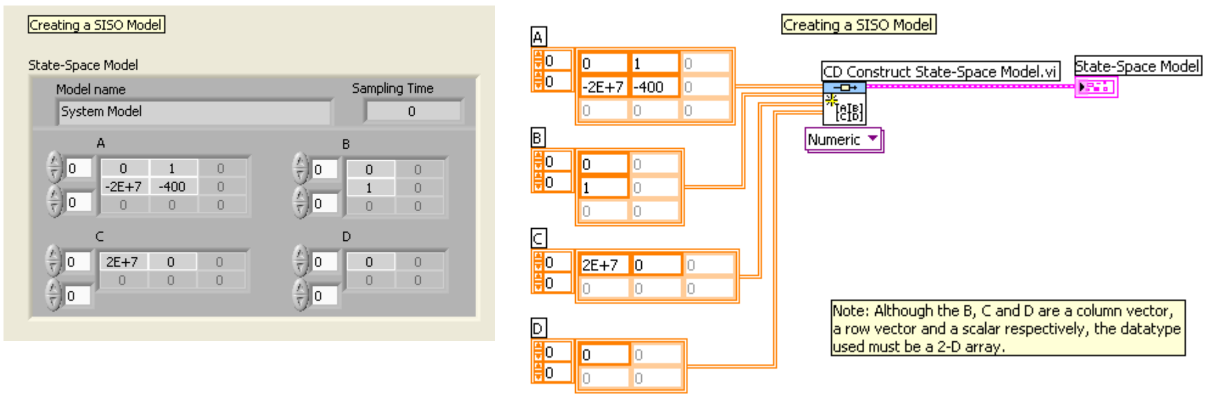
Front Panel:



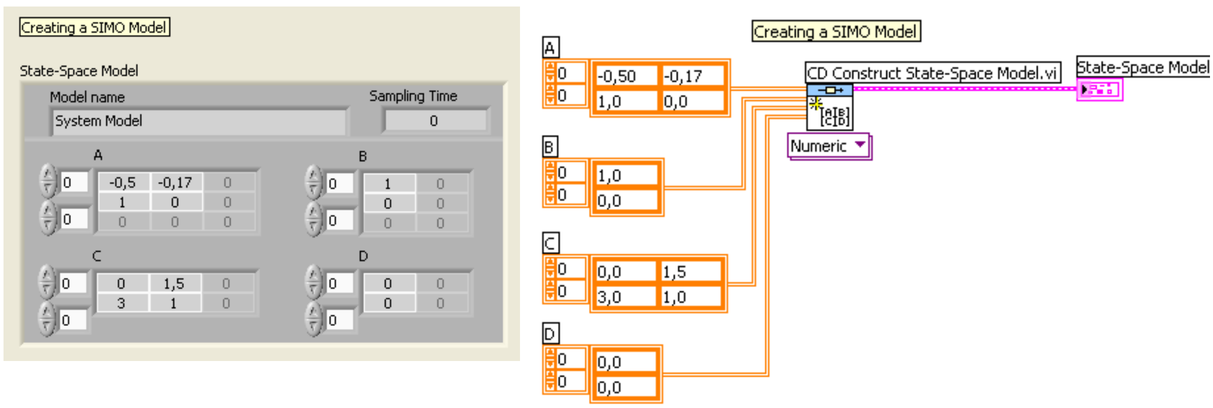
The “**CD Draw State-Space Equation.vi**” can be used to see a graphical representation of the State-space model.

Example: Create SISO/MIMO State-Space models

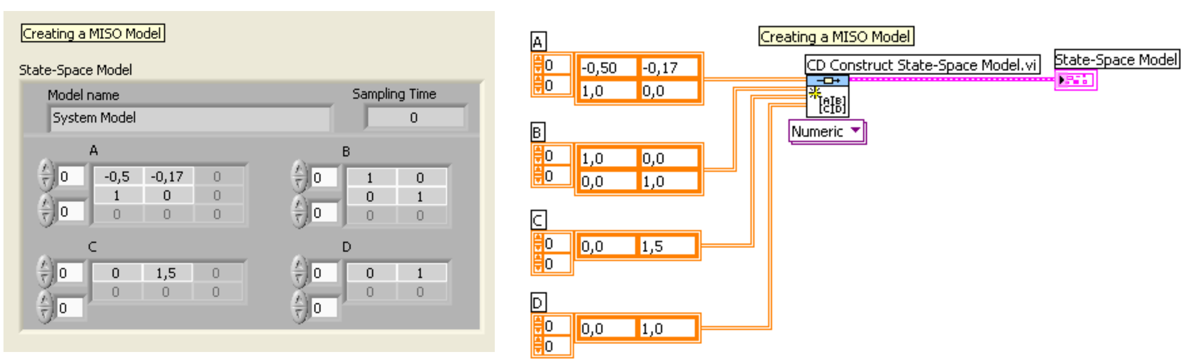
SISO Model (Single Input, Single Output):



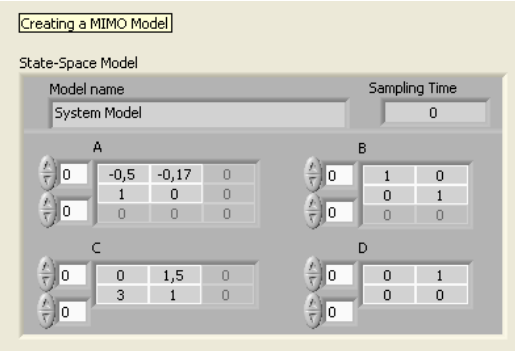
SIMO Model (Single Input, Multiple Output):

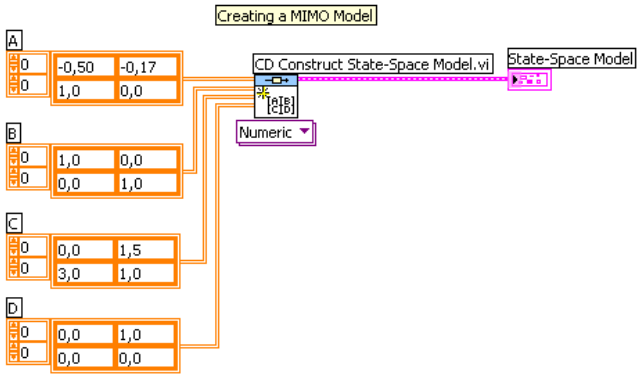


MISO Model (Multiple Input, Single Output):



MIMO Model (Multiple Input, Multiple Output):





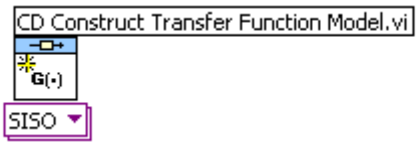
[End of Example]

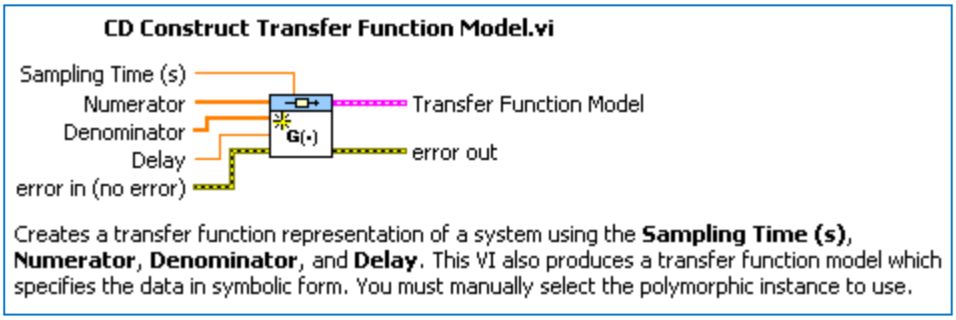
3.2 Transfer functions

Given the following Transfer function:

$$H(s) = \frac{numerator}{denominator} = \frac{b_0 + b_1 s + b_2 s^2 + \cdots}{a_0 + a_1 s + a_2 s^2 + \cdots}$$

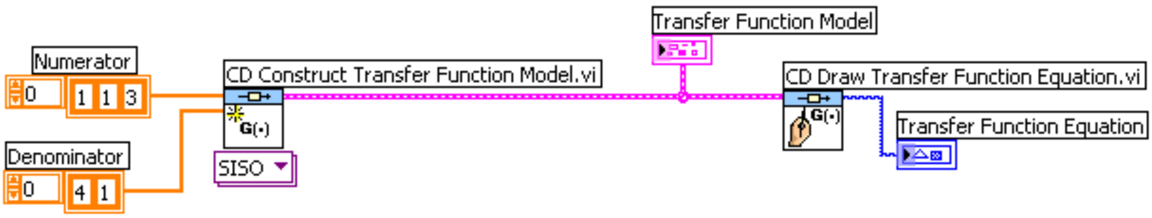
In LabVIEW we use the "**CD Construct Transfer Function Model.vi**" to create a Transfer Function:



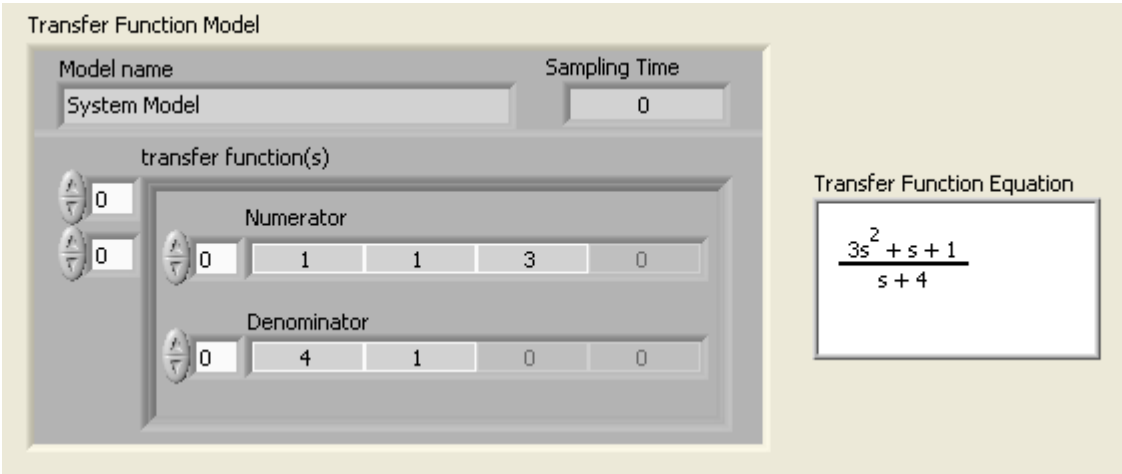


Example: Transfer Function

Block Diagram:



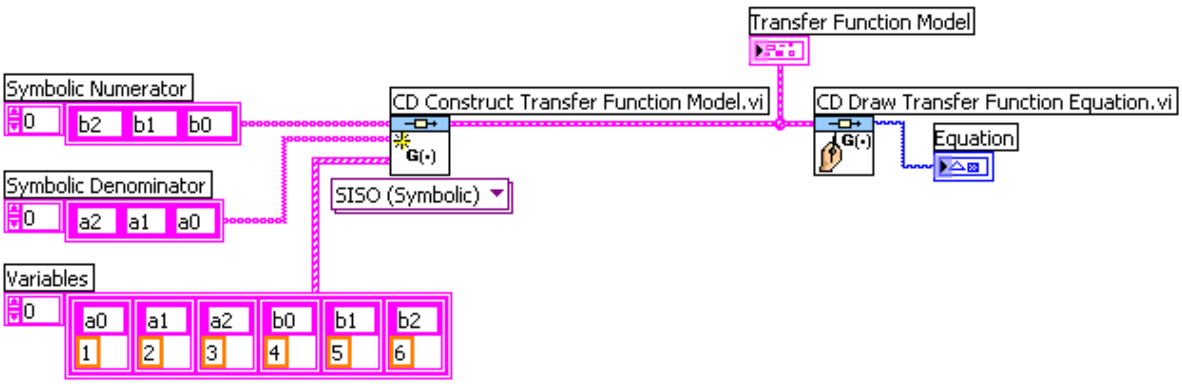
Front Panel:



[End of Example]

Example: Transfer Function with Symbolic values

Block Diagram:



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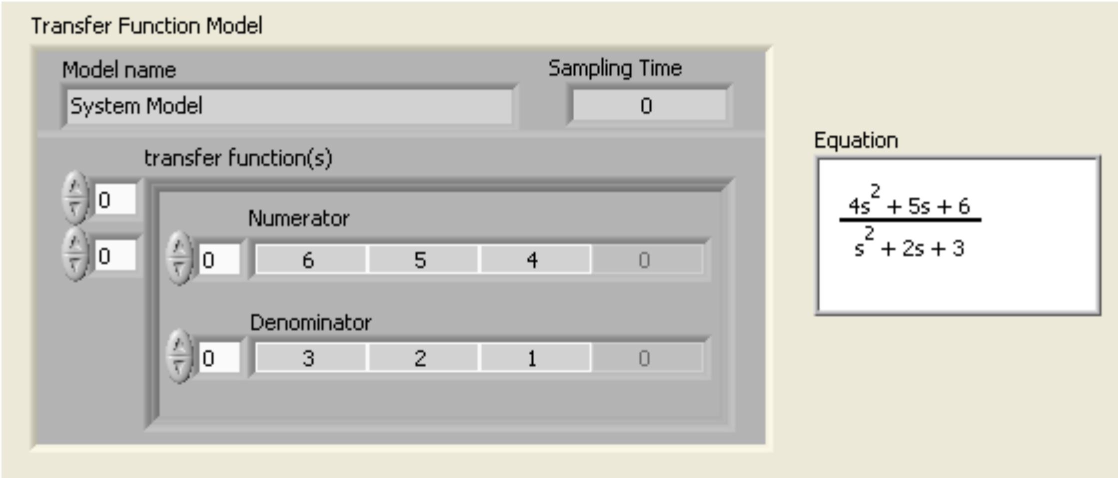


[End of Example]

3.2.1 commonly used transfer functions

For commonly used transfer functions we can use the "**CD Construct Special TF Model.vi**":

1.order system:

The transfer function for a 1. order system is as follows:

$$\boxed{H(s) = \frac{K}{Ts + 1}e^{-\tau s}}$$

Where

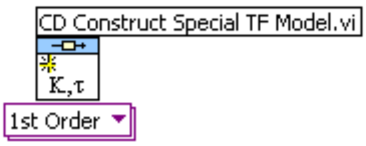
K is the gain

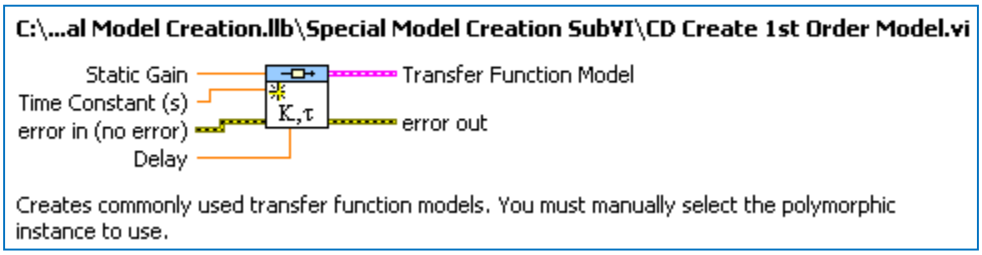
T is the Time constant

τ is the Time delay

Select the 

polymorphic instance on the "CD Construct Special TF Model.vi":





2.order system:

The transfer function for a 2. order system is as follows:

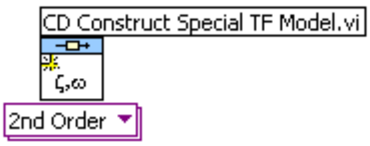
$$\boxed{H(s) = \frac{K{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2} = \frac{K}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\frac{s}{\omega_0} + 1}}$$

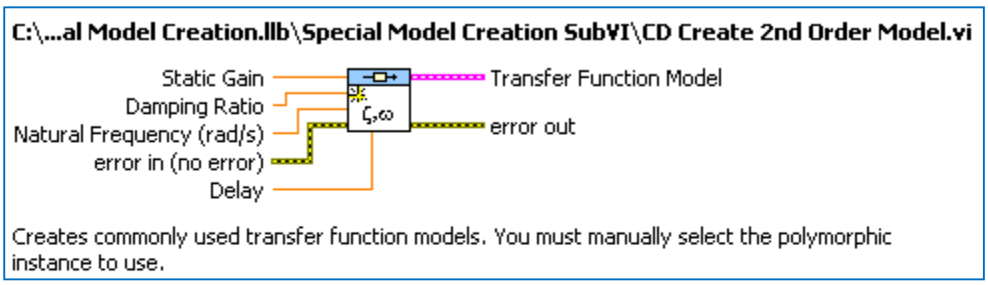
Where

- K is the gain
- ζ zeta is the relative damping factor
- ω_0[rad/s] is the undamped resonance frequency.

Select the 

polymorphic instance on the “CD Construct Special TF Model.vi”:





Time delay as a Pade’ approximation:

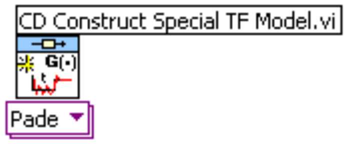
Time-delays are very common in control systems. The Transfer function of a time-delay is:

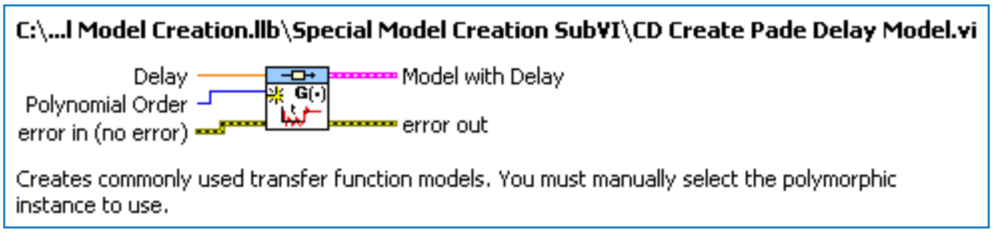
$$\boxed{H(s) = e^{-\tau s}}$$

In some situations it is necessary to substitute $e^{-\tau s}$ with an approximation, e.g., the Padé-approximation:

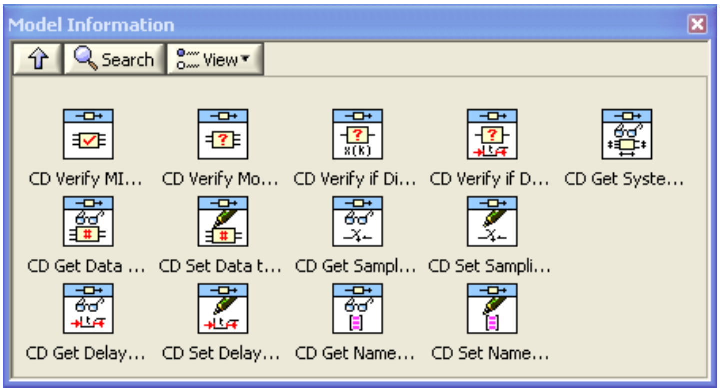
$$\boxed{e^{-\tau s} \approx \frac{1 - k_1 s + k_2 s^2 + \cdots \pm k_n s^n}{1 + k_1 s + k_2 s^2 + \cdots + k_n s^n}}$$

Select the  polymorphic instance on the "CD Construct Special TF Model.vi":



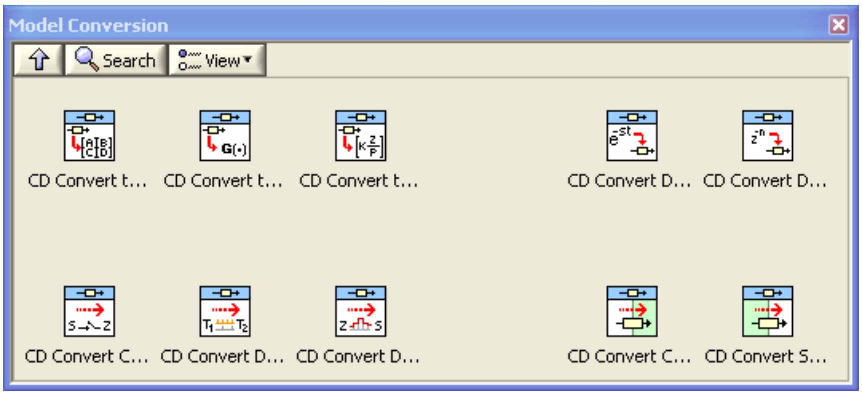


"Model Information" Subpalette:



→ Use the Model Information VIs to obtain or set parameters, data, and names of a system model. Model information includes properties such as the system delay, system dimensions, sampling time, and names of inputs, outputs, and states.

"Model Conversion" Subpalette:



→ Use the Model Conversion VIs to convert a system model from one representation to another, from a continuous-time to a discrete-time model, or from a discrete-time to a continuous-time model. You also can use the Model Conversion VIs to convert a control design model into a simulation model or a simulation model into a control design model.

Some of the most used VIs in the "Model Conversion" subpalette are:

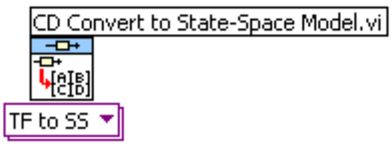
 **CD Convert to State-Space Model.vi**

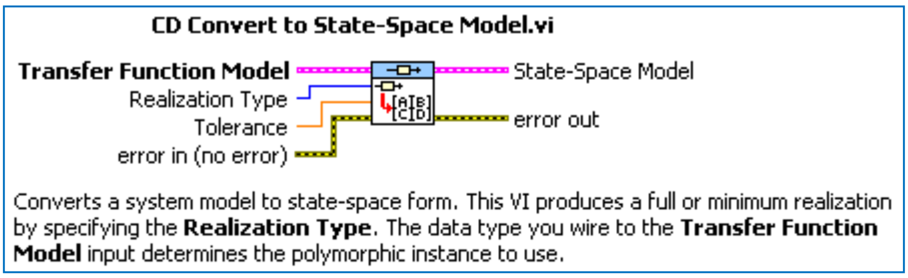
 **CD Convert to Transfer Function Model.vi**

 **CD Convert Continuous to Discrete.vi**

These VIs and some others are explained below.

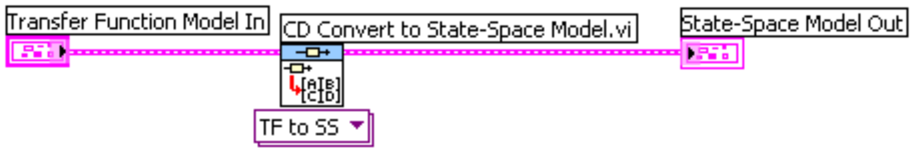
Convert to State-Space Models:



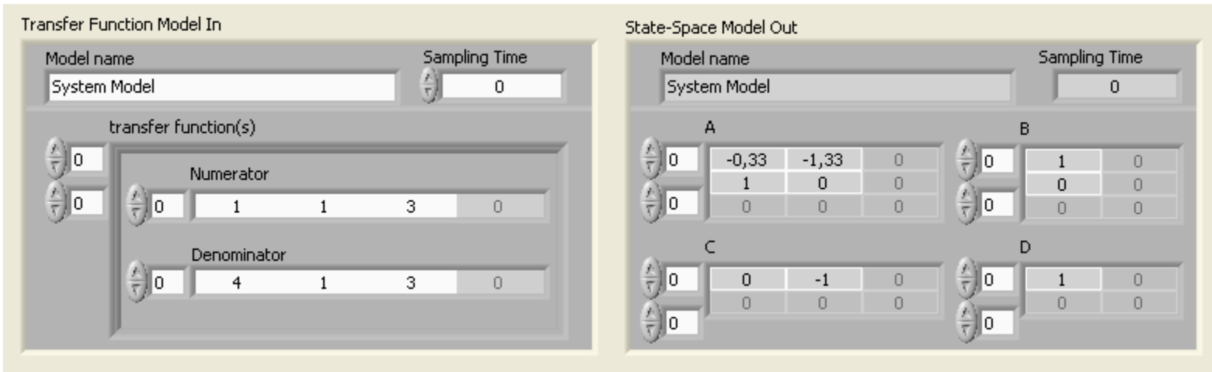


Example: Convert from Transfer function to State-Space model

Block Diagram:

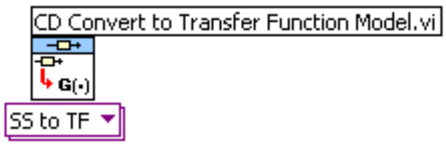


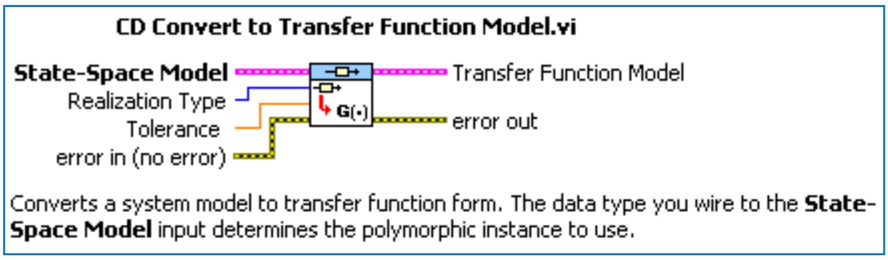
Front Panel:



[End of Example]

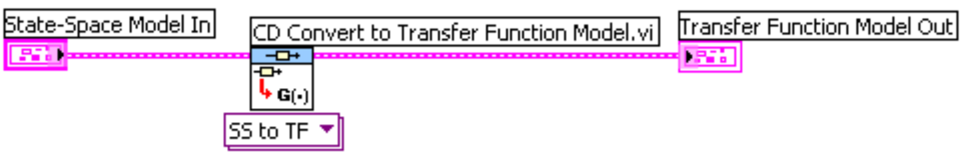
<u>Convert to Transfer Functions:</u>



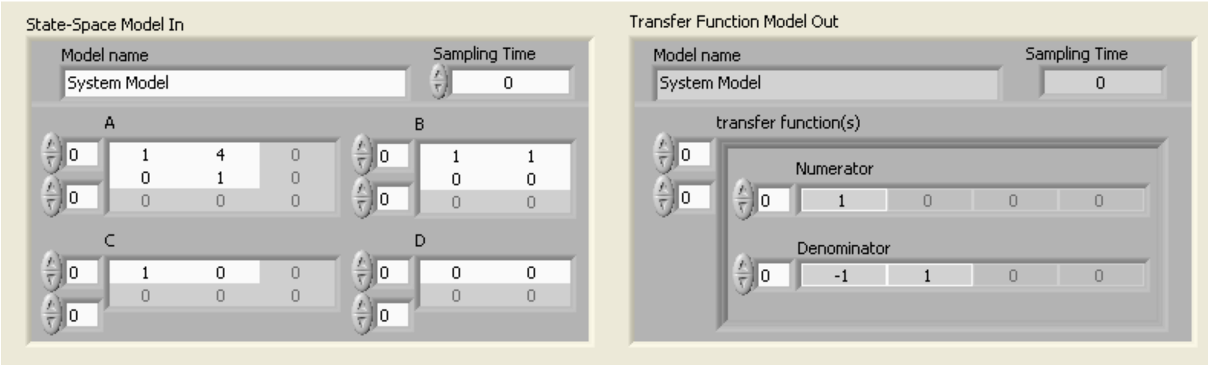


Example: Convert from State-Space model to Transfer Function

Block Diagram:

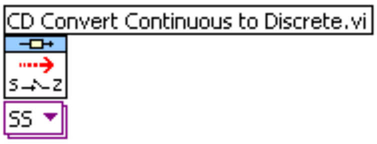


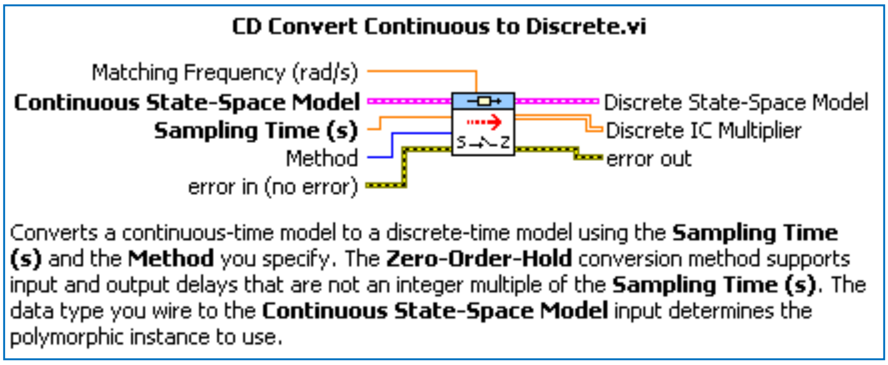
Front Panel:



[End of Example]

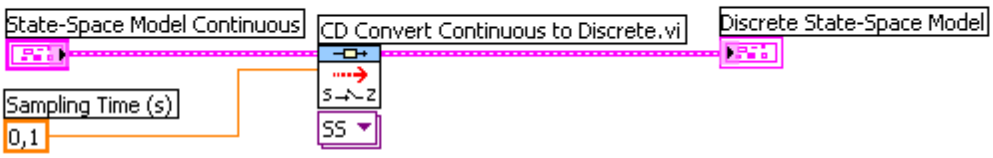
<u>Convert Continuous to Discrete Model:</u>



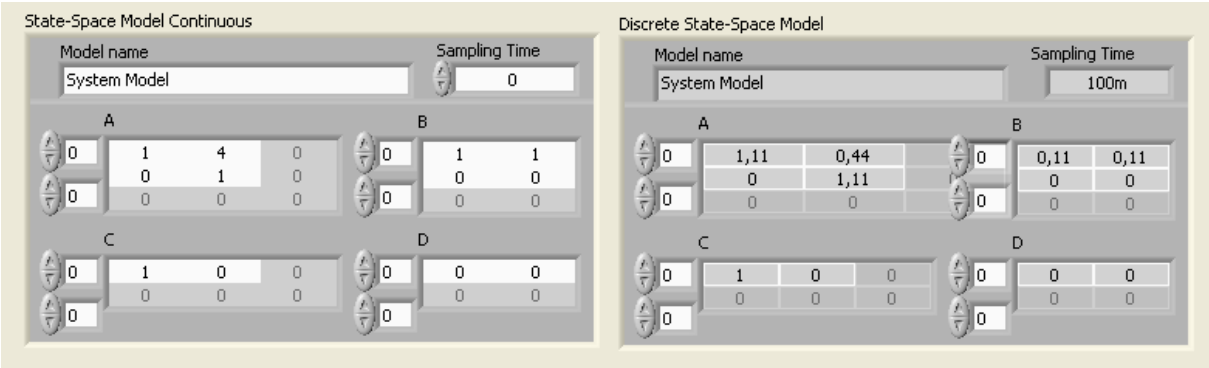


Example: Convert from Continuous State-Space model to Discrete State-Space model

Block Diagram:

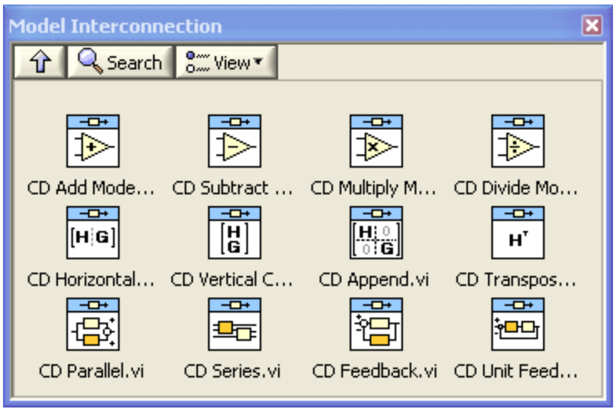


Front Panel:



[End of Example]

“Model Interconnection” Subpalette:



→ Use the Model Interconnection VIs to perform different types of linear system interconnections. You can build a large system model by connecting smaller system models together.

4 Introduction to System Identification and Estimation

This Tutorial will go through the basic principles of System identification and Estimation and how to implement these techniques in LabVIEW.

The following methods will be discussed in the next chapters:

State Estimation:

- Kalman Filter
- Observers

System Identification:

- Parameter Estimation and the Least Square Method (LS)
- Sub-space methods/Black-Box methods
- Polynomial Model Estimation: ARX/ARMAX model Estimation

The next chapters will go through the basic theory and show how it could be implemented in LabVIEW and MathScript.

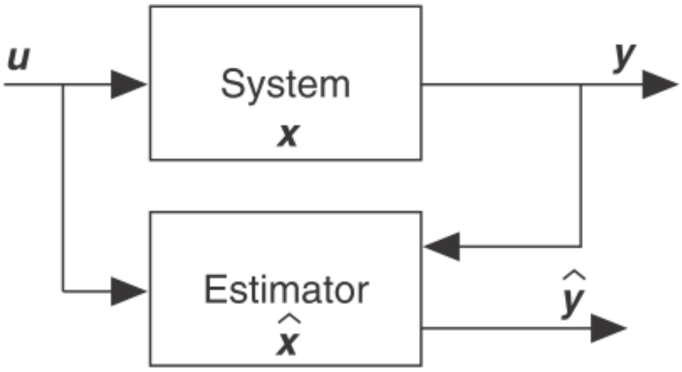
Part II: Estimation

5 State Estimation with Kalman Filter

Kalman Filter is a commonly used method to estimate the values of state variables of a dynamic system that is excited by stochastic (random) disturbances and stochastic (random) measurement noise.

The Kalman Filter is a state estimator which produces an optimal estimate in the sense that the mean value of the sum of the estimation errors gets a minimal value.

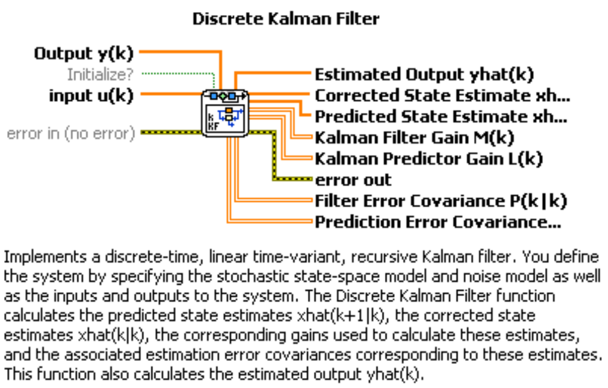
Below we see a sketch of how a Kalman Filter is working:

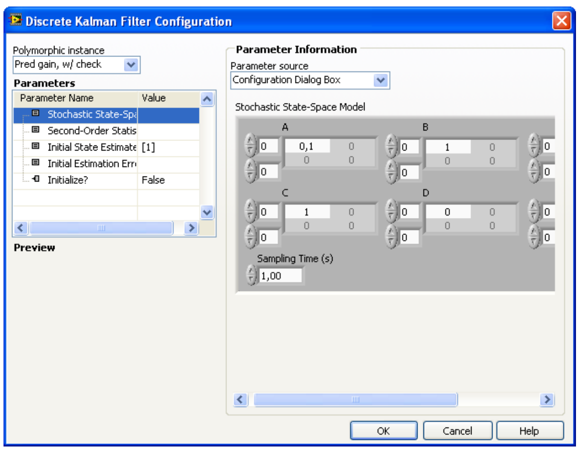


The estimator (model of the system) runs in parallel with the system (real system or model). The measurement(s) is used to update the estimator.

LabVIEW Control Design and Simulation Module have lots of functionality for State Estimation using Kalman Filters. The functionality will be explained in detail in the next chapters.

Below we see the Discrete Kalman Filter implementation in LabVIEW:





The Kalman Filter for nonlinear models is called the "**Extended Kalman Filter**".

5.1 State-Space model

Given the continuous linear state space-model:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

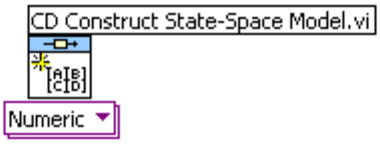
Or given the discrete linear state space-model

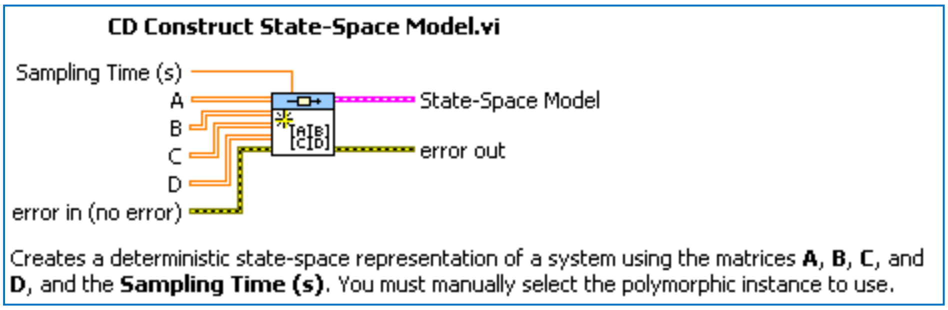
$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k + Du_k$$

LabVIEW:

In LabVIEW we may use the "**CD Construct State-Space Model.vi**" to create a State-space model:

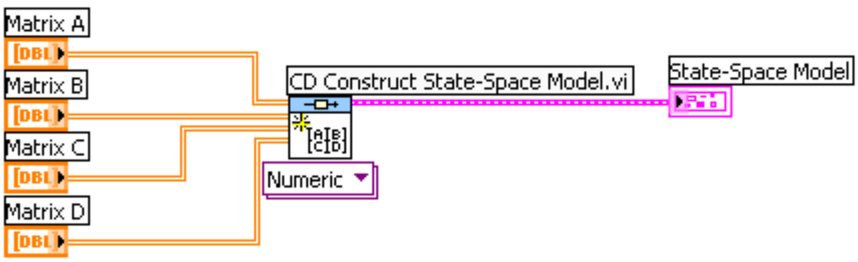




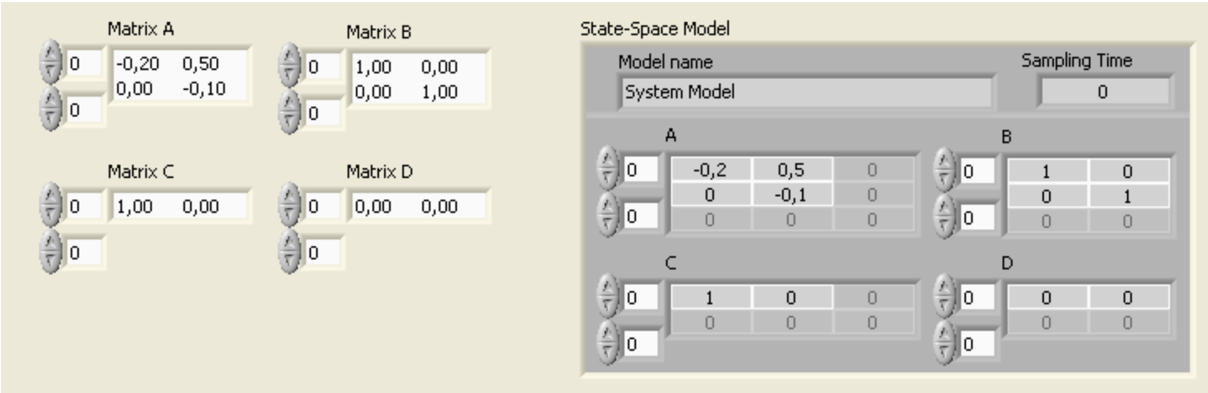
Note! If you specify a discrete State-space model you have to specify the Sampling Time.

LabVIEW Example: Create a State-space model

Block Diagram:



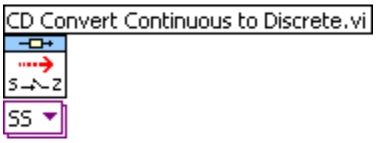
The matrices A, B, C and D may be defined on the Front Panel like this:



[End of Example]

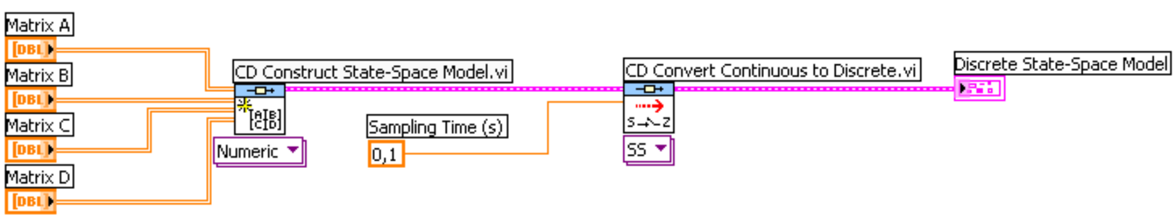
<u>Discretization:</u>

If you have a continuous model and want to convert it to the discrete model, you may use the VI "**CD Convert Continuous to Discrete.vi**" in LabVIEW:



LabVIEW Functions Palette: Control Design & Simulation → Control Design → Model Conversion → CD Convert Continuous to Discrete.vi

LabVIEW Example: Convert from Continuous to Discrete model



Note! We have to specify the Sampling Time.

[End of Example]

<u>MathScript:</u>

Use the **ss** function in MathScript to define your model (or **tf** if you have a transfer function). Use the **c_to_d** function to convert a continuous model to a discrete model.

MathScript Example:

Given the following State-space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The following MathScript Code creates this model:

```
c=1;
m=1;
k=1;
A = [0 1; -k/m -c/m];
B = [0; 1/m];
C = [1 0];
ssmodel = ss(A, B, C)
```

If you want to find the discrete model use the **c_to_d** function:

```
Ts=0.1 % Sampling Time
discretemodel = c_to_d(ssmodel, Ts)
```

[End of Example]

5.2 Observability

A necessary condition for the Kalman Filter to work correctly is that the system for which the states are to be estimated, is observable. Therefore, you should check for Observability before applying the Kalman Filter.

The Observability matrix is defined as:

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

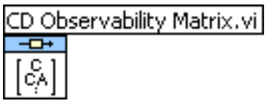
Where n is the system order (number of states in the State-space model).

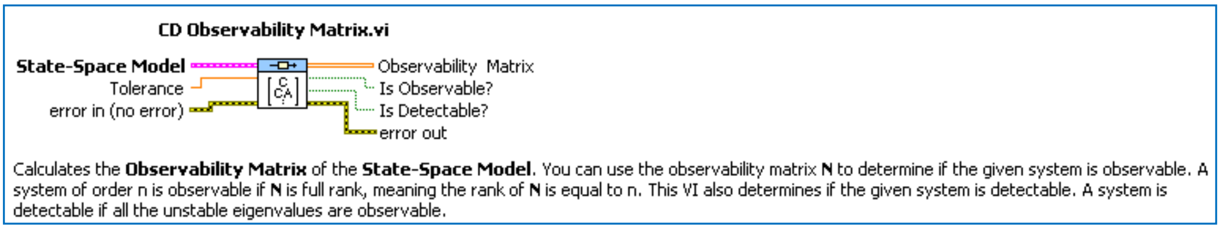
→ A system of order n is observable if O is full rank, meaning the rank of O is equal to n.

LabVIEW:

The LabVIEW Control Design and Simulation Module have a VI (**Observability Matrix.vi**) for finding the Observability matrix and check if a states-pace model is Observable.

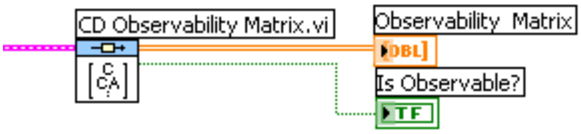
LabVIEW Functions Palette: Control Design & Simulation → Control Design → State-Space Model Analysis → CD Observability Matrix.vi





Note! In LabVIEW N is used as a symbol for the Observability matrix.

LabVIEW Example: Check for Observability



[End of Example]

MathScript:

In MathScript you may use the **obsvmx** function to find the Observability matrix. You may then use the **rank** function in order to find the rank of the Observability matrix.

MathScript Example:

The following MathScript Code check for Observability:

```
% Check for Observability:
O = obsvmx (discretemodel)
r = rank(O)
```

[End of Example]

5.3 Introduction to the State Estimator

Continuous Model:

Given the continuous linear state space model:

$$\dot{x} = Ax + Bu + Gw$$

$$y = Cx + Du + Hv$$

or in general:

$$\dot{x} = f(x, u) + Gw$$

$$y = g(x, u) + Hv$$

Discrete Model:

Given the discrete linear state space model:

$$x_{k+1} = Ax_k + Bu_k + Gw_k$$

$$y_k = Cx_k + Du_k + Hv_k$$

or in general:

$$x_{k+1} = f(x_k, u_k) + Gw_k$$

$$y_k = g(x_k, u_k) + Hv_k$$

Where v is uncorrelated white process noise with zero mean and covariance matrix Q and w is uncorrelated white measurements noise with zero mean and covariance matrix, i.e. such that

$$Q = E\{ww^T\}$$

$$R = E\{vv^T\}$$

$E\{w\}$ is the expected value or mean of the process noise vector.

$E\{v\}$ is the expected value or mean of the measurement noise vector.

It is normal to let Q and R be diagonal matrices:

$$Q = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & q_{nn} \end{bmatrix}, R = \begin{bmatrix} r_{11} & 0 & 0 & 0 \\ 0 & r_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & r_{rr} \end{bmatrix}$$

G is the process noise gain matrix, and you normally set G equal to the Identity matrix I:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H is the measurement noise gain matrix, and you normally set $H = 0$

State Estimator:

The **state estimator** is given by:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K(y - \hat{y})$$

$$\hat{y}_k = C\hat{x}_k$$

Where K is the Kalman Filter Gain

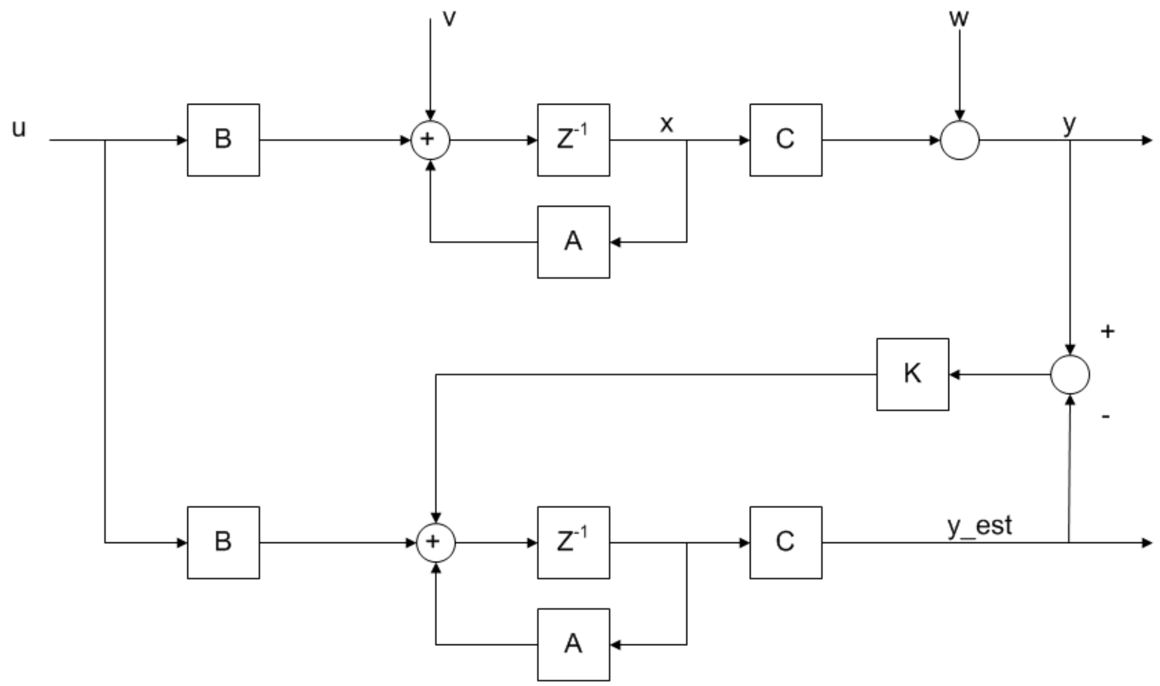
It can be found that:

$$K = XC^T Q^{-1}$$

Where X is the solution to the Riccati equation. It is common to use the steady state solution of the Riccati equation (Algebraic Riccati Equation), i.e., $\dot{X} = 0$.

Note! Q and R is used as tuning/weighting matrices when designing the Kalman Filter Gain K.

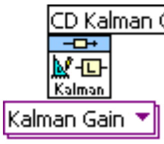
Below we see a **Block Diagram** of a (discrete) Kalman Filter/State Estimator:

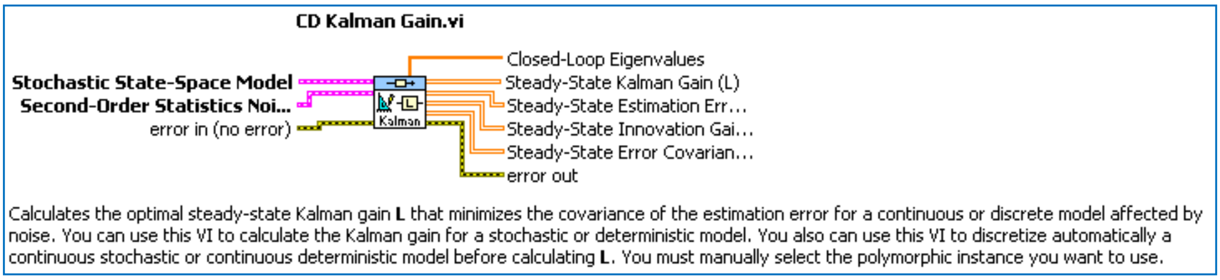


LabVIEW:

In LabVIEW Design and Simulation Module we use the "**CD Kalman Gain.vi**" in order to find K:

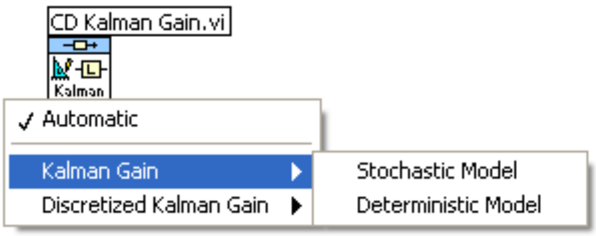
LabVIEW Functions Palette: Control Design & Simulation → Control Design → State Feedback Design → CD Kalman Gain.vi

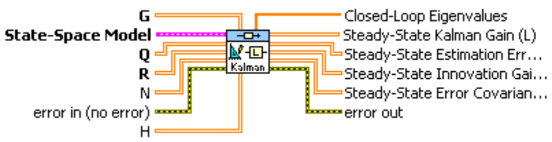
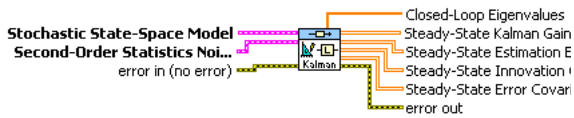
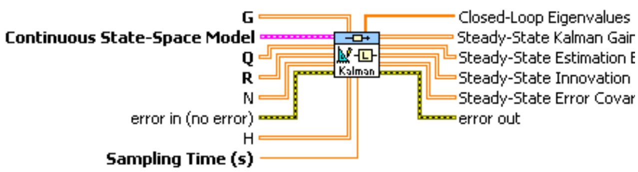
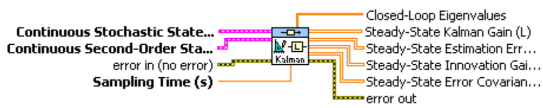




Note! In LabVIEW L is used as a symbol for the Kalman Filter Gain matrix.

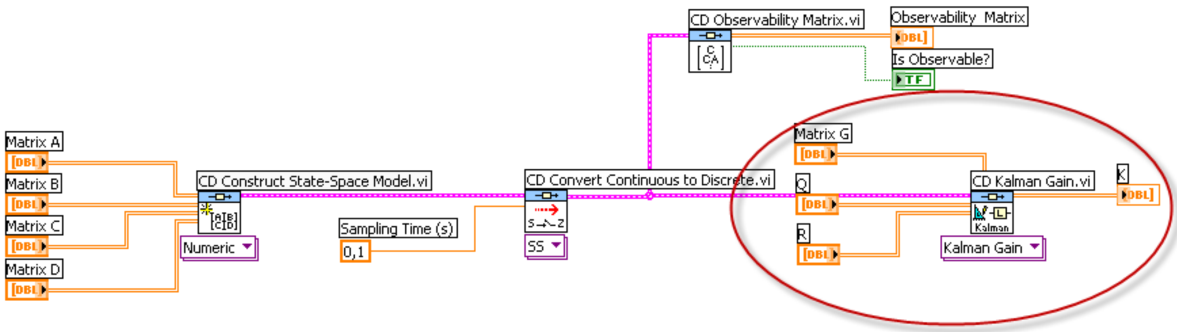
Note! The "Kalman Filter Gain.vi" is polymorphic, depending on what kind of model (deterministic/stochastic or continuous/discrete) you wire to this VI, the inputs changes automatically or you may use the polymorphic selector below the VI:



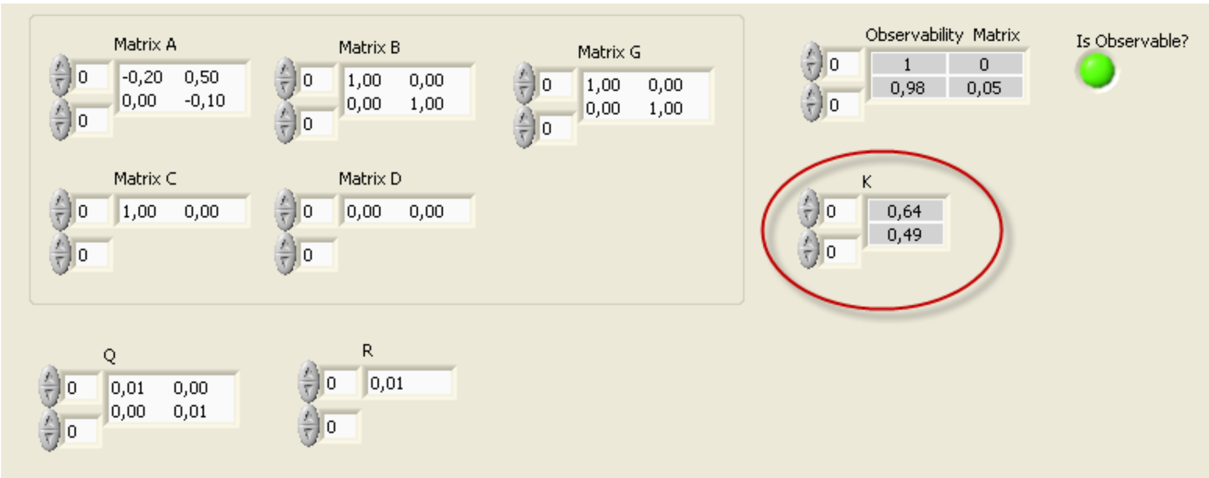
Deterministic and Continuous:	Stochastic and Continuous:
	
Deterministic and Discrete:	**Stochastic and Discrete:**
	

LabVIEW Example: Find the Kalman Gain

Block Diagram:



Front Panel:



[End of Example]

MathScript:

Use the functions **kalman**, **kalman_d** or **lqe** to find the Kalman gain matrix.

Function	Description	Example
kalman	Calculates the optimal steady-state Kalman gain **K** that minimizes the covariance of the state estimation error. You can use this function to calculate **K** for continuous and discrete system models.	`>>[SysKal, K]=kalman(ssmodel, Q, R)`

kalman_d	Calculates the optimal steady-state Kalman gain **K** that minimizes the covariance of the state estimation error. The input system and noise covariance are based on a continuous system. All outputs are based on a discretized system, which is based on the sample rate **Ts**.	`[SysKalDisc, K]=kalman_d(ssmodel, Q, R, Ts)`
lqe	Calculates the optimal steady-state estimator gain matrix **K** for a continuous state-space model defined by matrices **A** and **C**.	`K=lqe(A,G,C,Q,R)`

MathScript Example: Find Kalman Gain using the Kalman function

MathScript Code for finding the steady state Kalman Gain:

```
% Define the State-space model:
c=1;
m=1;
k=1;
A = [0 1; -k/m -c/m];
B = [0; 1/m];
C = [1 0];
ssmodel = ss(A, B, C);
% Discrete model:
Ts=0.1; % Sampling Time
discretemodel = c_to_d(ssmodel, Ts);
% Check for Observability:
O = obsvmx(discretemodel);
r = rank(O);
% Find Kalman Gain
Q=[0.01 0; 0 0.01];
R=[0.01];
[SysKal, K]=kalman(discretemodel, Q, R);
K
```

The Output is:

```
K =
     0.64435
     0.10759
```

[End of Example]

MathScript Example: Find Kalman Gain using the Kalman function

MathScript Code for finding the steady state Kalman Gain:

```
% Define the State-space model:
c=1;
m=1;
k=1;
A = [0 1; -k/m -c/m];
B = [0; 1/m];
G=[1 0 ; 0 1];
C = [1 0];

% Find Kalman Gain
Q=[0.01 0; 0 0.01];
R=[0.01];

K=lqe(A,G,C,Q,R)
```

The Output is:

```
K =
     0.86121
    -0.12916
```

[End of Example]

5.4 State Estimation

LabVIEW Control Design and Simulation Module have built-in functionality for State Estimation using the Kalman Filter.

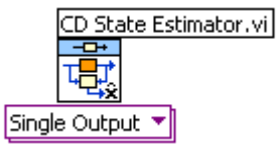
In the next section we will create our own Kalman Filter State Estimation algorithm.

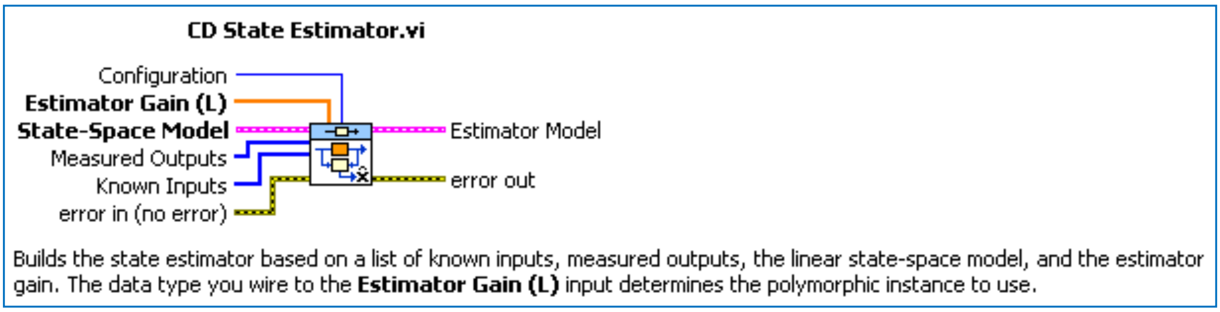
LabVIEW:

There are different functions and VIs for finding the State Estimation using the Kalman Filter.

CD State Estimator.vi:

LabVIEW Functions Palette: Control Design & Simulation → Control Design → State Feedback Design → CD State Estimator.vi





Below we show an example of how to use the CD State Estimator.vi in LabVIEW.

LabVIEW Example: State Estimator Simulation

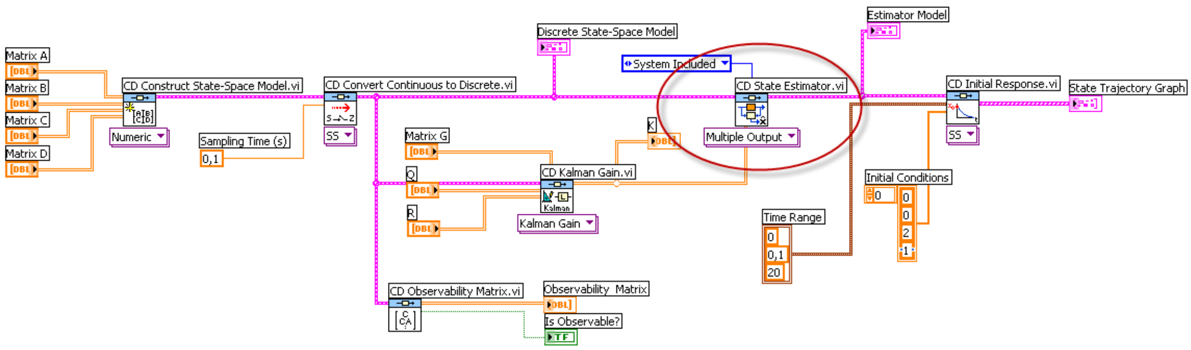
Given the following model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -0.2 & 0.5 \\ 0 & -0.1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$

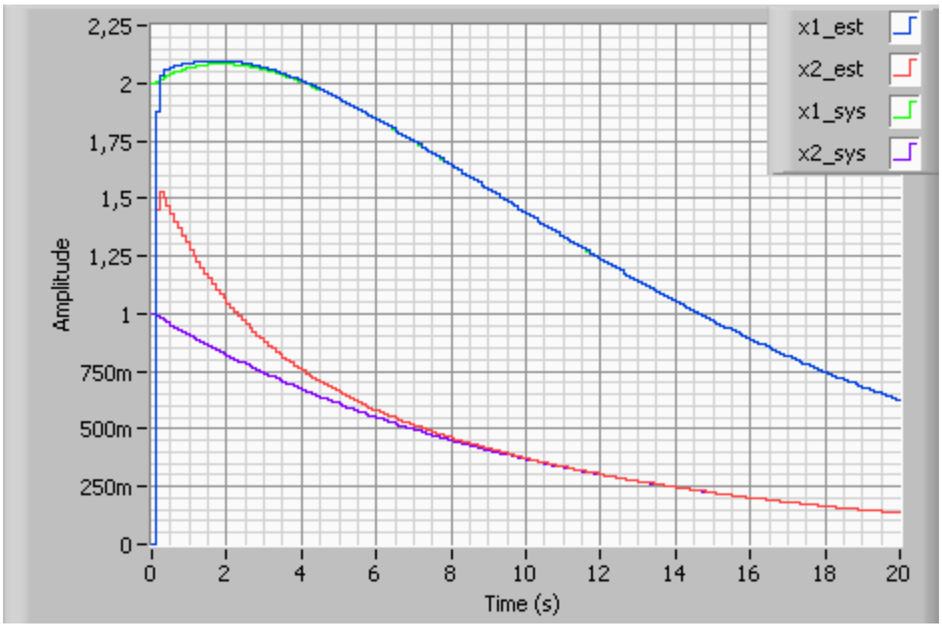
We will use the "**CD State Estimator.vi**" in LabVIEW.

Block Diagram becomes as follows:



Note! We have used "**CD Initial Response.vi**" for plotting the response. The VI is located in the LabVIEW Functions Palette: Control Design & Simulation → Control Design → Time Response → CD Initial Response.vi

The result becomes as follows:



We see the estimates are good.

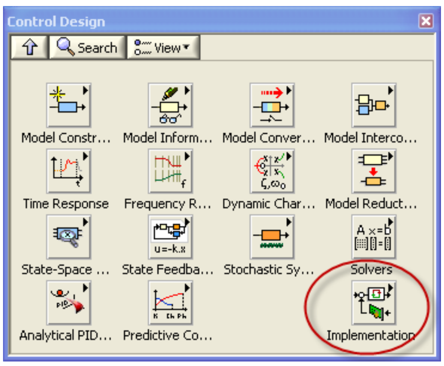
MathScript:

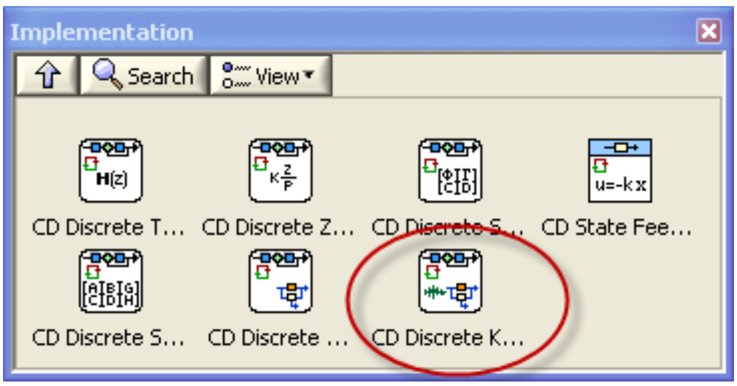
In MathScript we may use the built-in **estimator** function.

5.5 LabVIEW Kalman Filter Implementations

LabVIEW Design and Simulation Module have several built-in versions of the Kalman Filter; here we will investigate some of them.

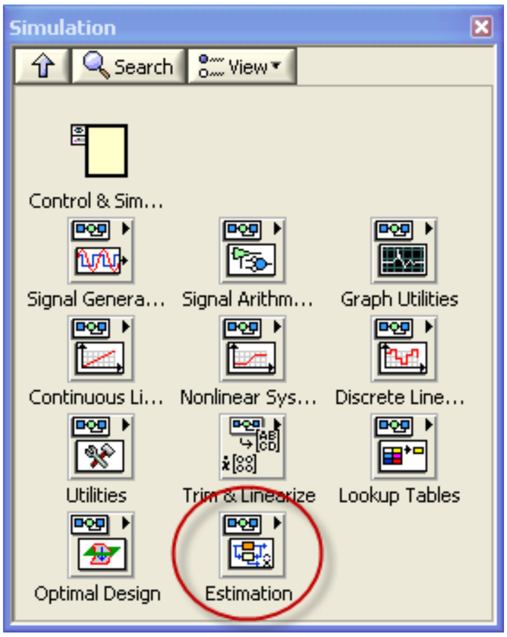
The Control Design → Implementation palette in LabVIEW:

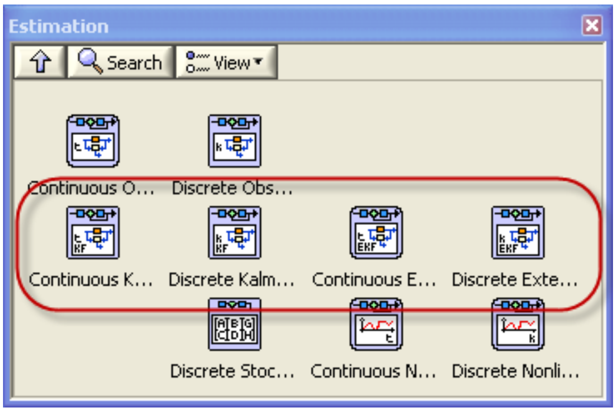




Here we have the "CD Discrete Kalman Filter".

Simulation → Estimation palette in LabVIEW:





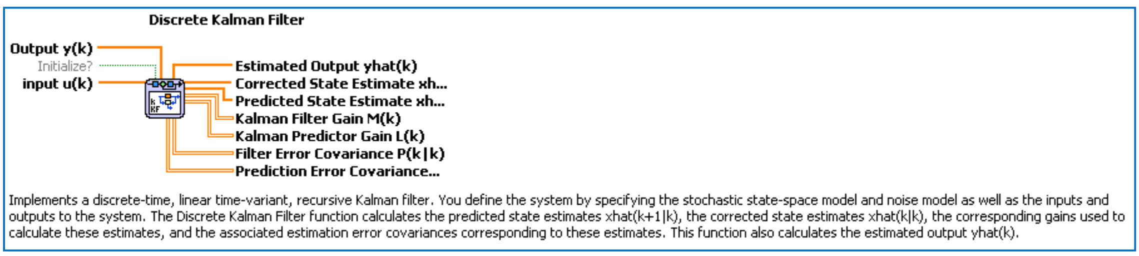
Here we have implementations for:

- Continuous Kalman Filter
- Continuous Extended Kalman Filter
- Discrete Kalman Filter
- Discrete Extended Kalman Filter

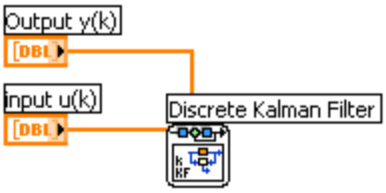
We will go through the "Discrete Kalman Filter" in detail and show some examples.

<u>Discrete Kalman Filter:</u>

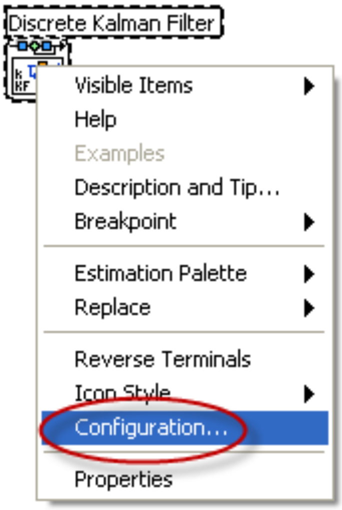
LabVIEW Functions Palette: Control Design & Simulation → Simulation → Estimation → Discrete Kalman Filter



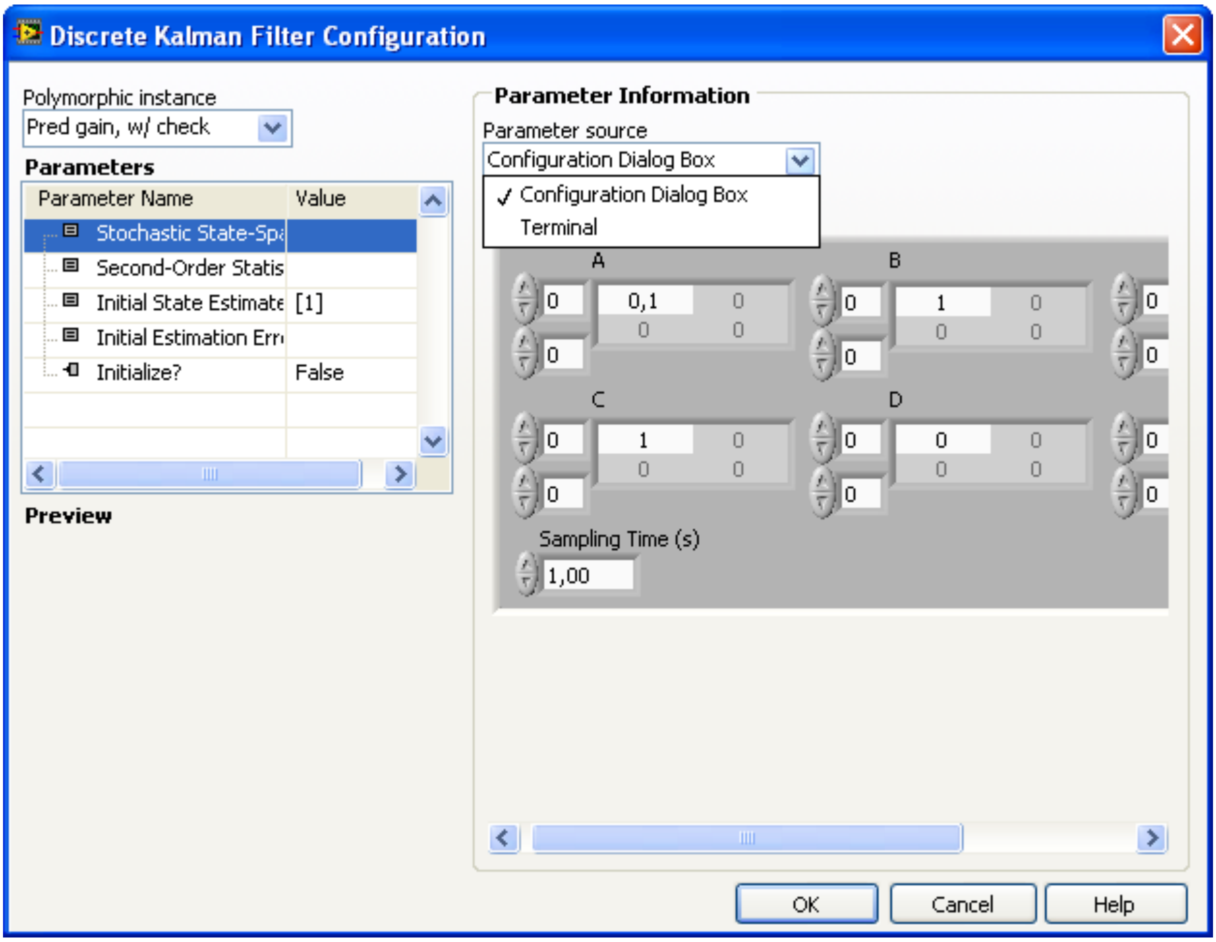
By default you need to wire the input (u) and output (y) vectors:



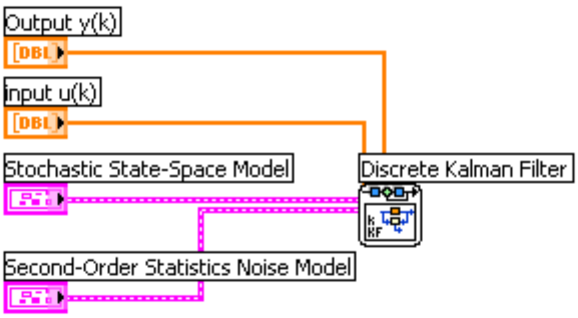
In order to Configure the block you right-click on it and select “Configuration…”



In the Configuration window you can enter your model parameters:



If you select “Terminal” in the “Parameter source” you may create your model in LabVIEW code like this:



LabVIEW Example: Discrete Kalman Filter

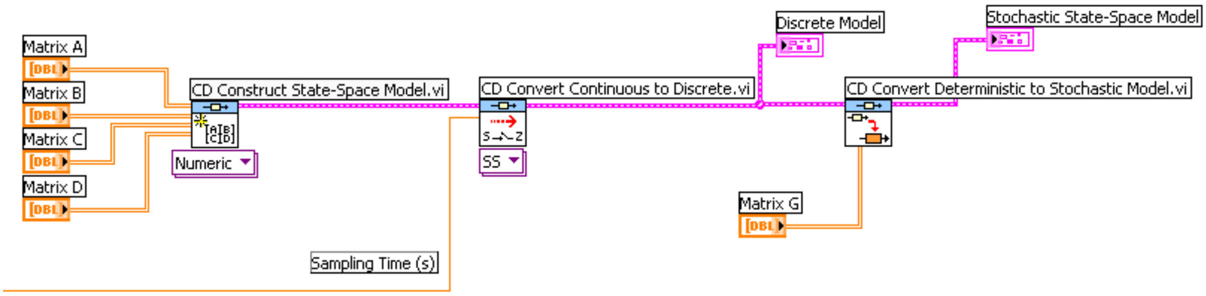
Given the following linear state-space model of a water tank:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -10 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0.02 \\ 0 \end{bmatrix}}_{B} u$$

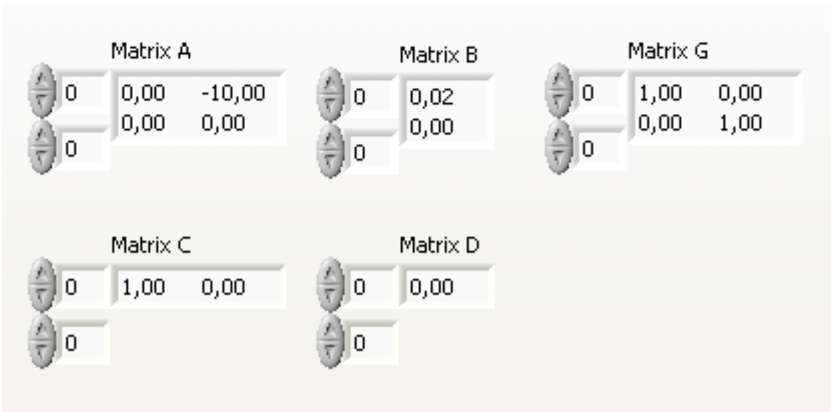
$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$

Where x_1 is the level in the tank, while x_2 is the outflow of the tank. Only the level x_1 is measured.

Step 1: First we create the model:

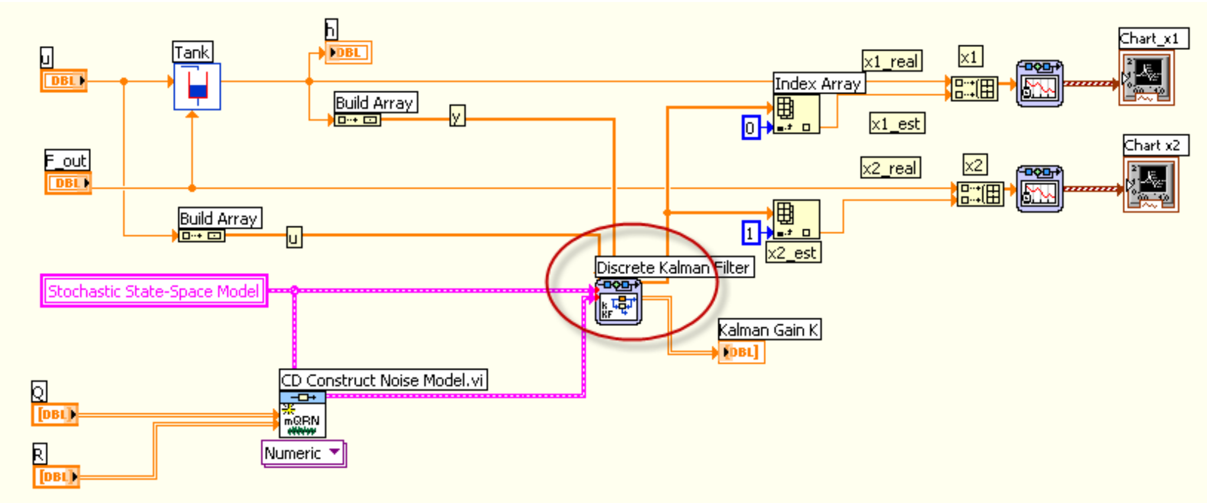


Where A, B, D and D is defined according to the state-space model above:



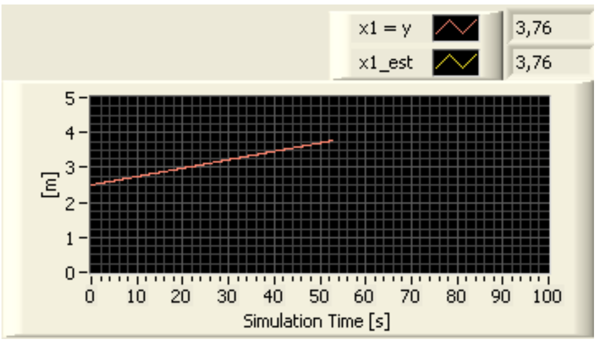
Note! The Discrete Kalman Filter function in LabVIEW requires a stochastic state-space model, so we have to create a stochastic state-space model or convert our state-space model into a stochastic state-space model as done in the LabVIEW code above.

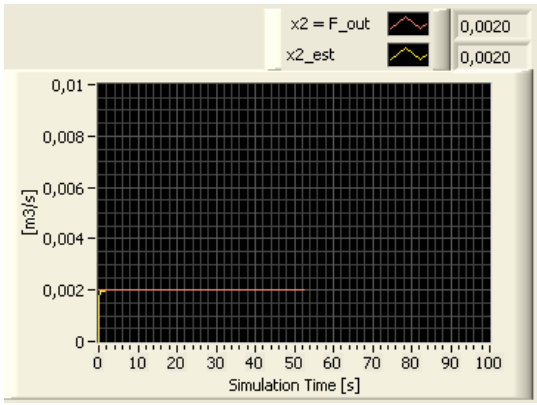
Step 2: Then we use the Discrete Kalman Filter function in LabVIEW on our model:



The Discrete Kalman Filter function also requires a Noise model, so we create a noise model from our Q and R matrices as done in the LabVIEW code above.

The results are as follows:





We see the result is very good.

[End of Example]

6 Create your own Kalman Filter from Scratch

In this chapter we will create our own Kalman Filter Algorithm from scratch.

6.1 The Kalman Filter Algorithm

LabVIEW Design and Simulation Module have several built-in versions of the Kalman Filter, but in this chapter we will create our own Kalman Filter algorithm.

Here is a step by step Kalman Filter algorithm which can be directly implemented in a programming language, such as LabVIEW. You may, e.g., implement it in standard LabVIEW code or a Formula Node in LabVIEW.

Pre Step: Find the steady state Kalman Gain K

K is time-varying, but you normally implement the steady state version of Kalman Gain K. Use the "**CD Kalman Gain.vi**" in LabVIEW or one of the functions **kalman**, **kalman_d** or **lqe** in MathScript.

Init Step: Set the initial Apriori (Predicted) state estimate

$$\bar{x}_0 = x_0$$

Step 1: Find Measurement model update

$$\bar{y}_k = g(\bar{x}_k, u_k)$$

For Linear State-space model:

$$\bar{y}_k = C\bar{x}_k + Du_k$$

Step 2: Find the Estimator Error

$$e_k = y_k - \bar{y}_k$$

Step 3: Find the Aposteriori (Corrected) state estimate

$$\hat{x}_k = \bar{x}_k + Ke_k$$

Where K is the Kalman Filter Gain. Use the steady state Kalman Gain or calculate the time-varying Kalman Gain.

Step 4: Find the Apriori (Predicted) state estimate update

$$\bar{x}_{k+1} = f(\hat{x}_k, u_k)$$

For Linear State-space model:

$$\bar{x}_{k+1} = A\hat{x}_k + Bu_k$$

Step 1-4 goes inside a loop in your program.

This is the algorithm we will implement in the example below.

6.2 Examples

LabVIEW Example: Kalman Filter algorithm

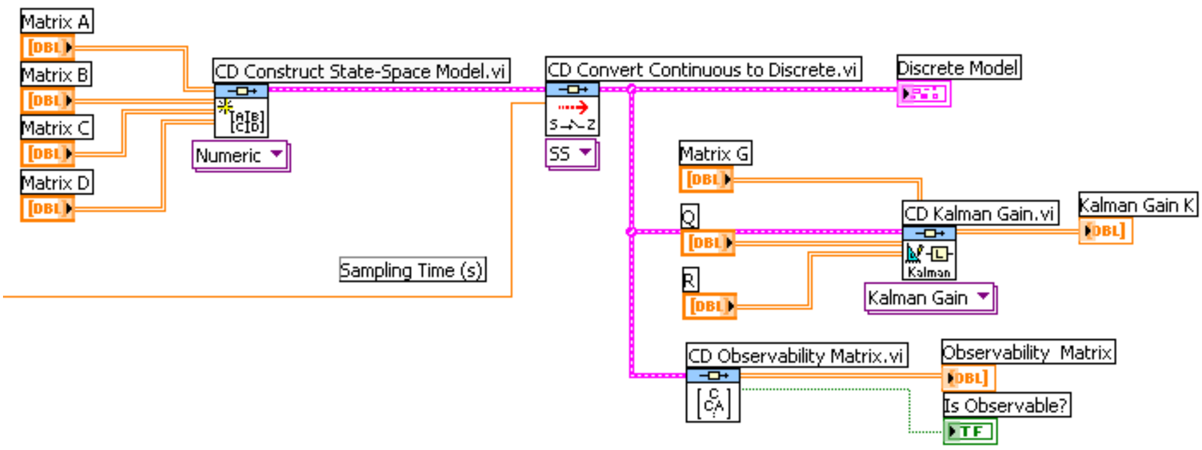
Given the following linear state-space model of a water tank:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -10 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0.02 \\ 0 \end{bmatrix}}_{B} u$$

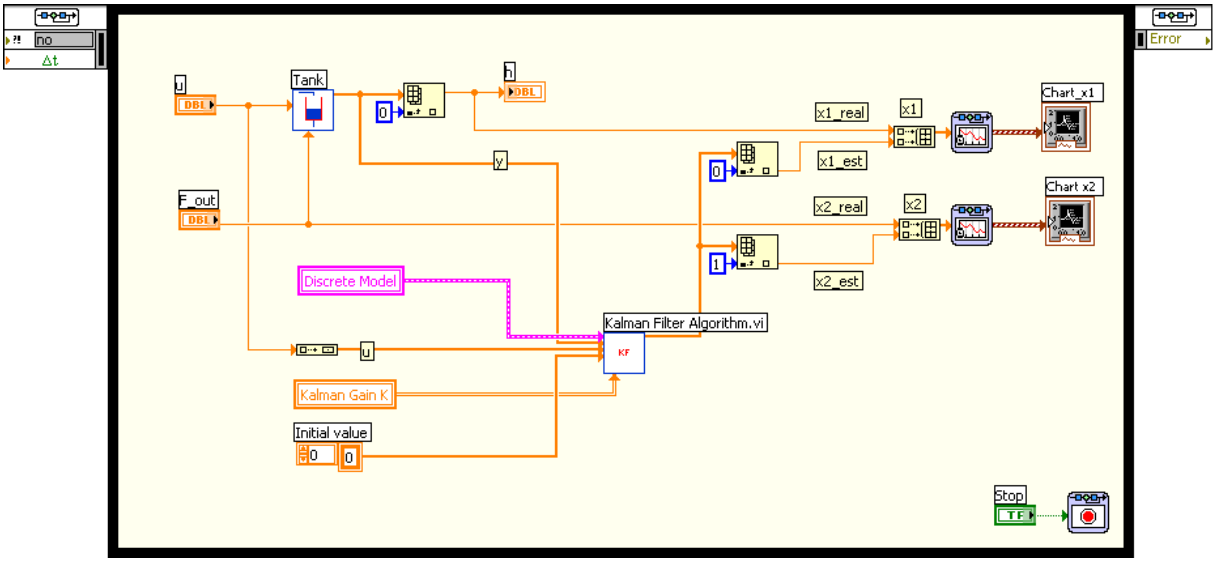
$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$

Where x_1 is the level in the tank, while x_2 is the outflow of the tank. Only the level x_1 is measured.

First we have to find the steady state Kalman Filter Gain and check for Observability:



Then we run the real process (or simulated model) in parallel with the Kalman Filter in order to find estimates for x_1 and x_2:



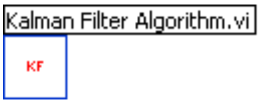
In this case we have used a **Simulation Loop**, but a While Loop will do the same.

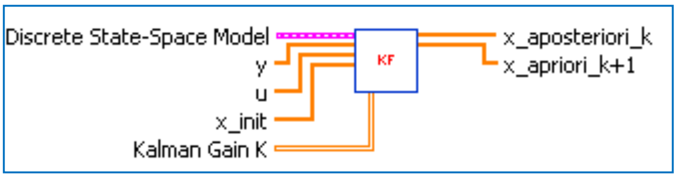
Blocks/SubVIs:

Real process/Simulated process:

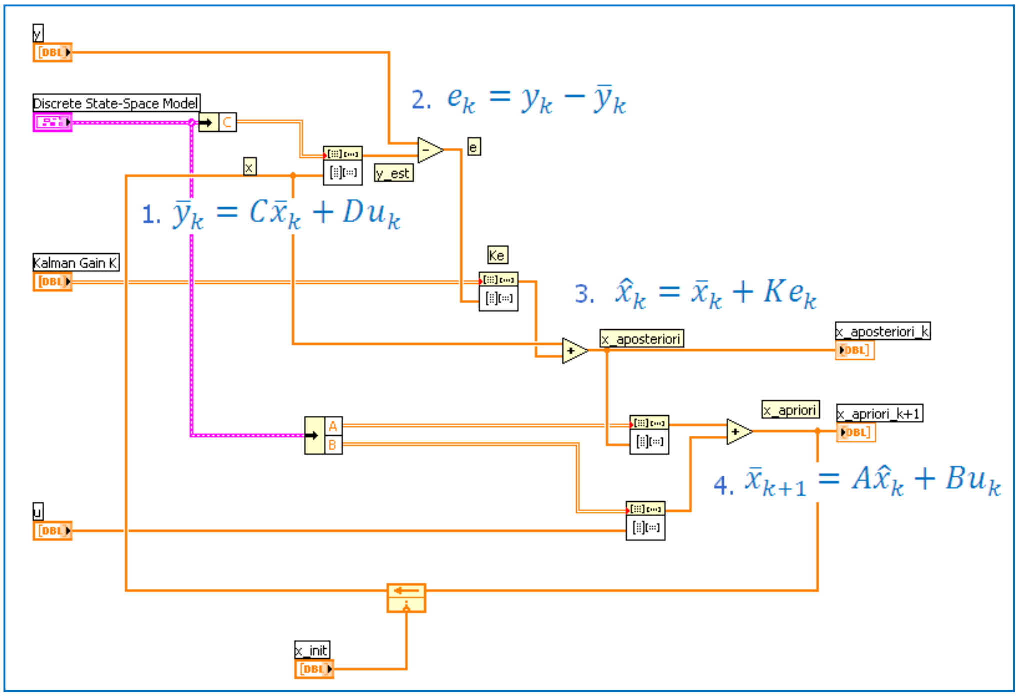
Here we either have a model of the system or read/write data from the real process using a DAQ card, e.g., USB-6008 from National Instruments.

Implementation of the Kalman Filter Algorithm:



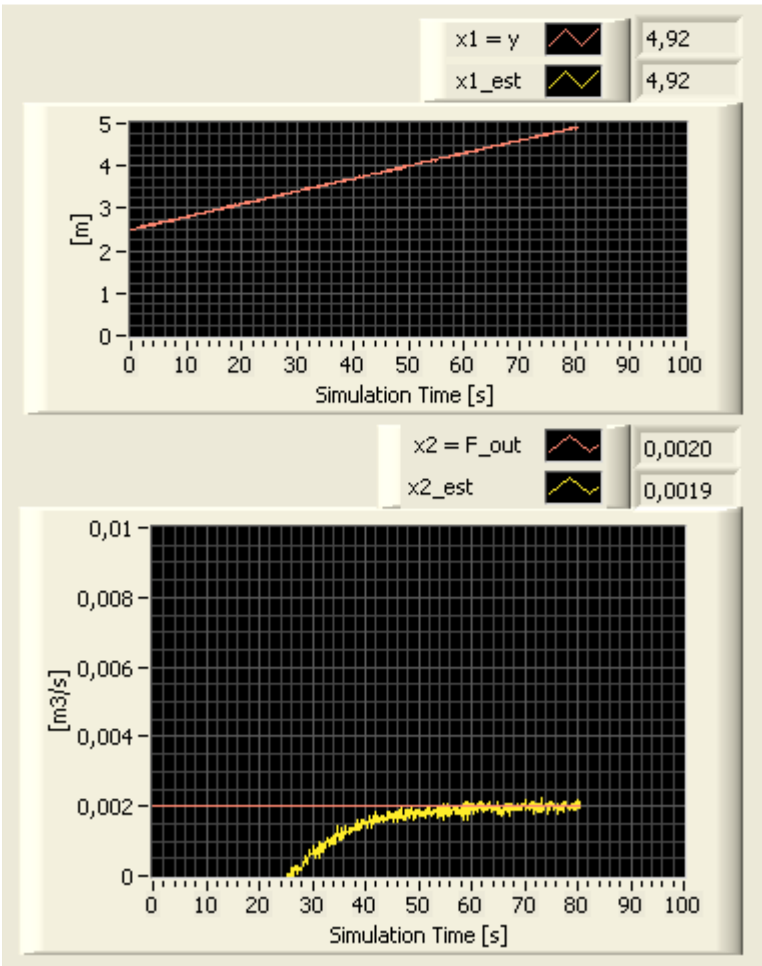


The Block Diagram is as follows:



This is a general implementation and will work for all linear discrete systems.

The results are as follows:



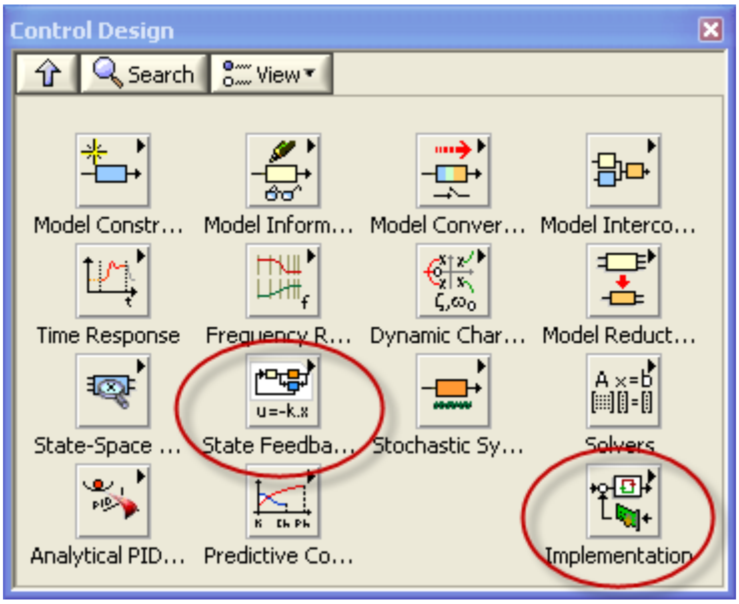
[End of Example]

7 Overview of Kalman Filter VIs

In LabVIEW there are several VIs and functions used for Kalman Filter implementations.

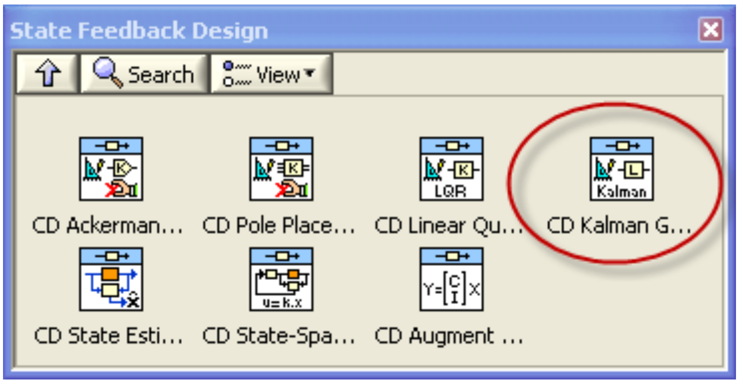
7.1 Control Design Palette

In the "**Control Design**" palette we find subpalettes for "**State Feedback Design**" and "**Implementation**":



7.1.1 State Feedback Design subpalette

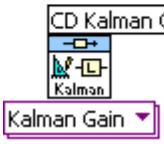
In the "**State Feedback Design**" subpalette we find VIs for calculation the Kalman Gain, etc.

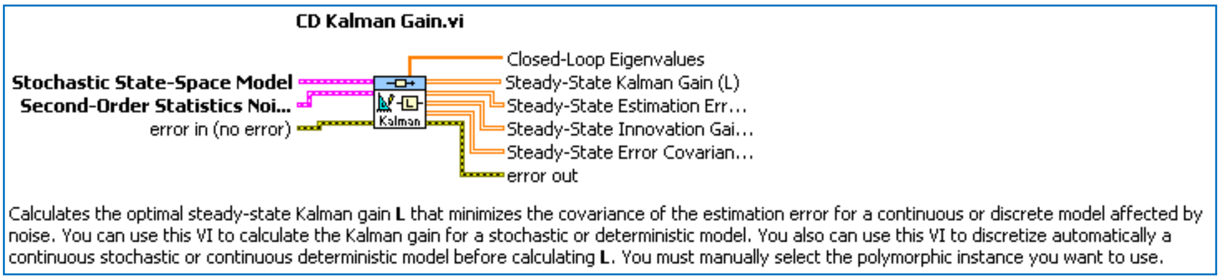


→ Use the State Feedback Design VIs to calculate controller and observer gains for closed-loop state feedback control or to estimate a state-space model. You also can use State

Feedback Design VIs to configure and test state-space controllers and state estimators in time domains.

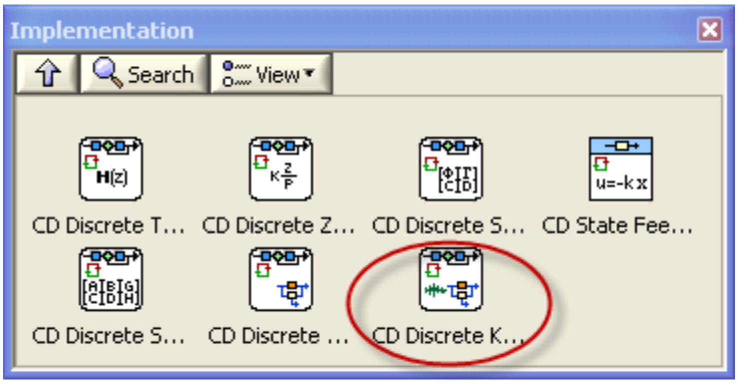
Kalman Filter Gain VI:





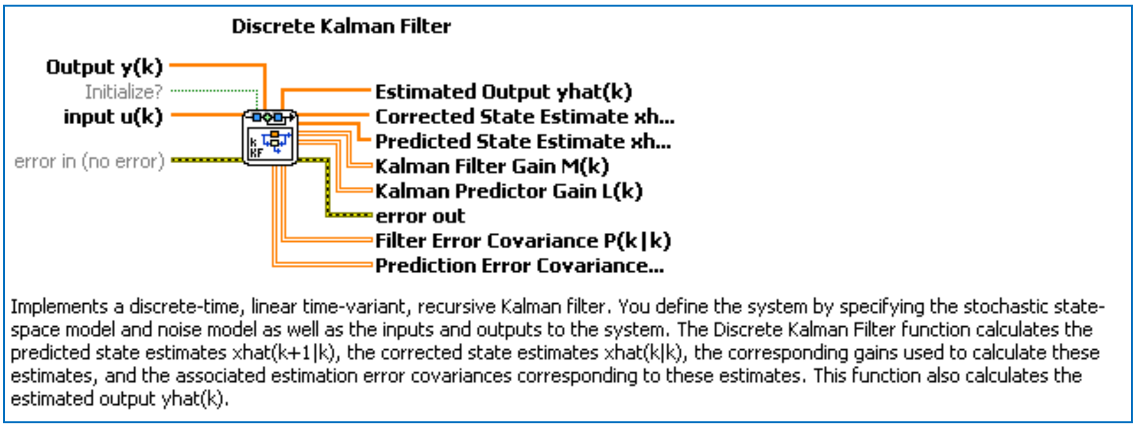
7.1.2 Implementation subpalette

In the “**Implementation**” subpalette we find VIs for implementing a discrete Observer and a discrete Kalman Filter.



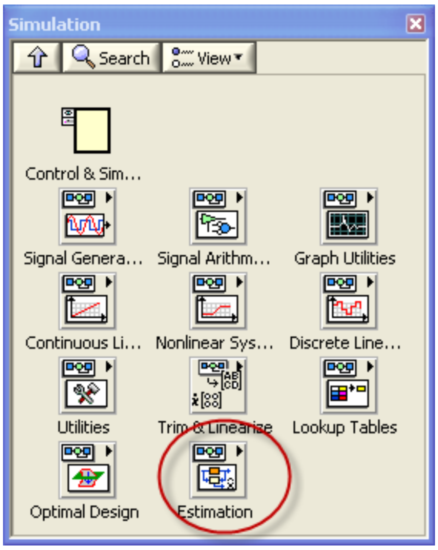
→ Use the Implementation VIs and functions to simulate the dynamic response of a discrete system model, deploy a discrete model to a real-time target, implement a discrete Kalman filter, and implement current and predictive observers.

Discrete Kalman Filter:



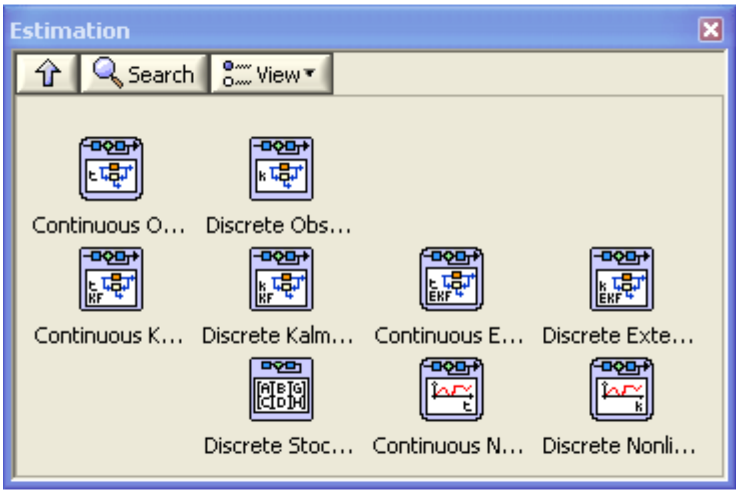
7.2 Simulation Palette

In the "**Simulation**" palette we find the "**Estimation**" subpalette:



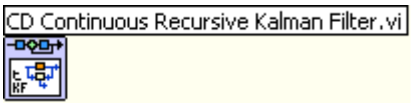
7.2.1 Estimation subpalette

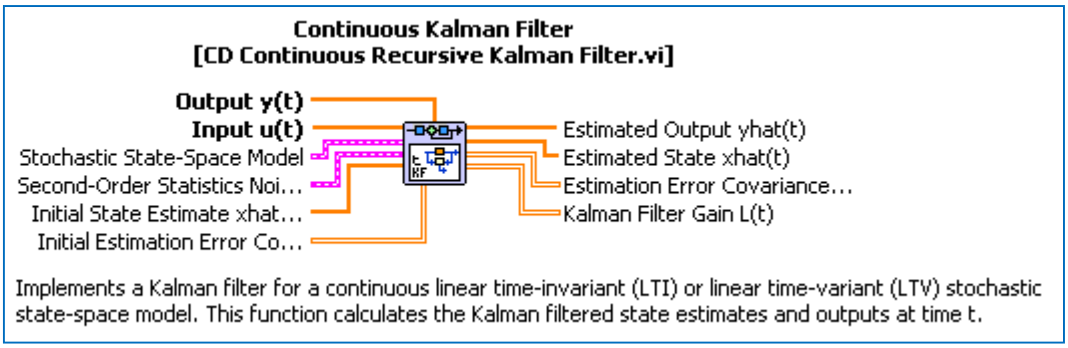
In the "**Estimation**" palette we find VIs for implementing a continuous/discrete Kalman Filter.

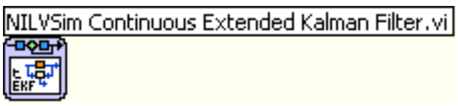


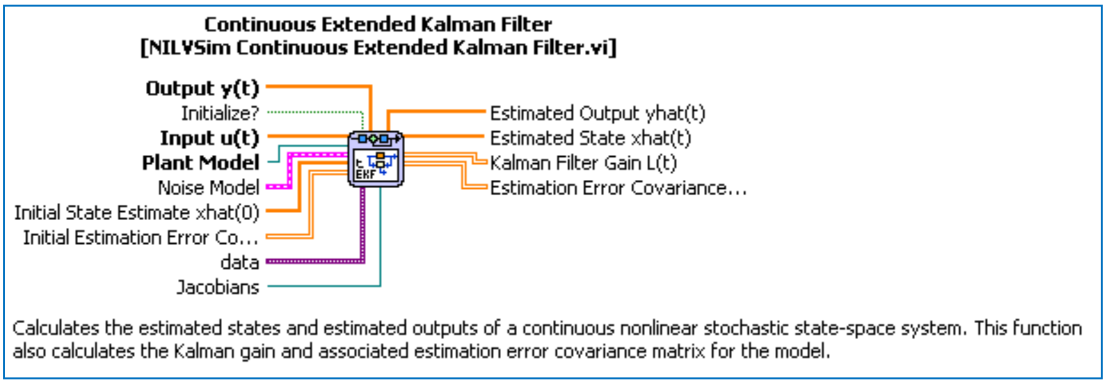
→ Use the Estimation functions to estimate the states of a state-space system. The state-space system can be deterministic or stochastic, continuous or discrete, linear or nonlinear, and completely or partially observable.

Continuous Kalman Filter VIs:

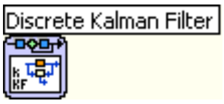


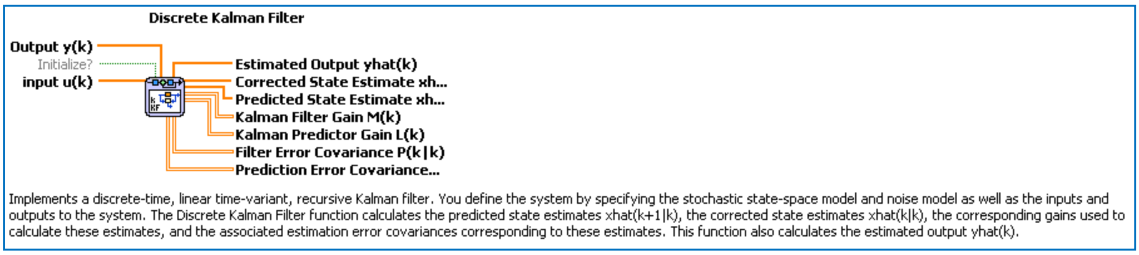


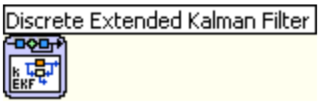


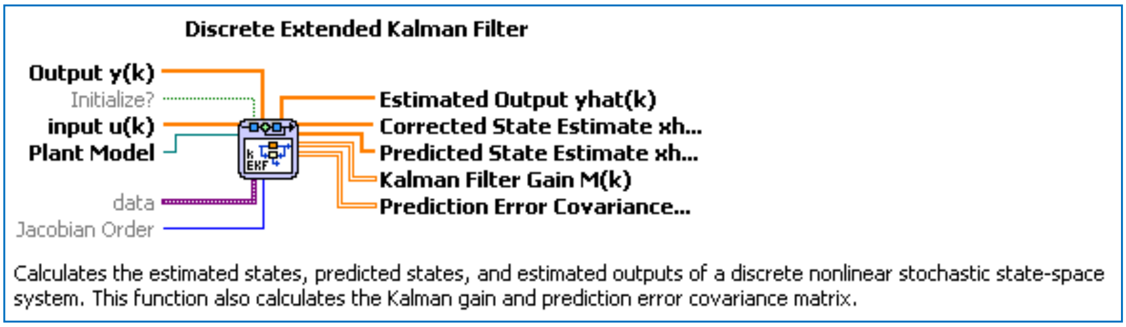


Discrete Kalman Filter VIs:







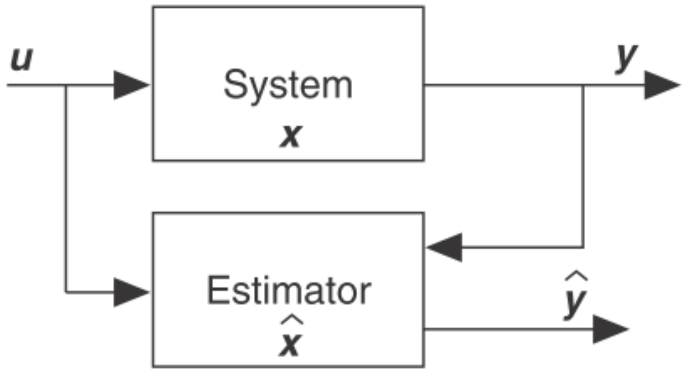


8 State Estimation with Observers in LabVIEW

Observers are an alternative to the Kalman Filter. An Observer is an algorithm for estimating the state variables in a system based on a model of the system. Observers have the same structure as a Kalman Filter.

In Observers you specify how fast and stable you want the estimates to converge to the real values, i.e., you specify the eigenvalues of the system. Based on the eigenvalues you will find the Observer gain K that is used to update the estimates.

One simple way to find the eigenvalues is to use the Butterworth eigenvalues from the Butterworth polynomial. When we have found the eigenvalues we can then use the Ackerman in order to find the Observer gain.



LabVIEW Control Design and Simulation Module have lots of functionality for State Estimation using Observers. The functionality will be explained in detail in the next chapters.

8.1 State-Space model

Given the continuous linear state space-model:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

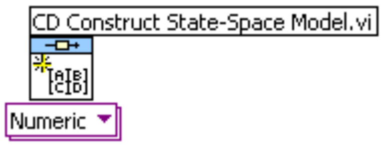
Or given the discrete linear state space-model

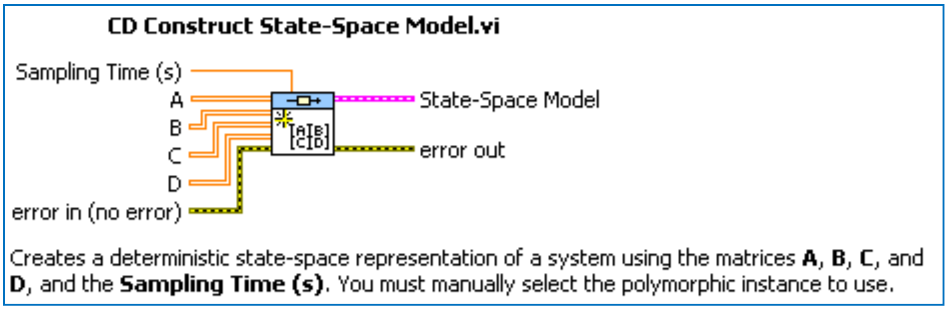
$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k + Du_k$$

LabVIEW:

In LabVIEW we may use the "**CD Construct State-Space Model.vi**" to create a State-space model:

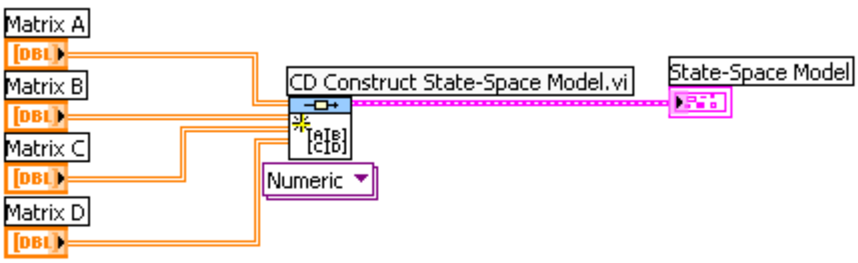




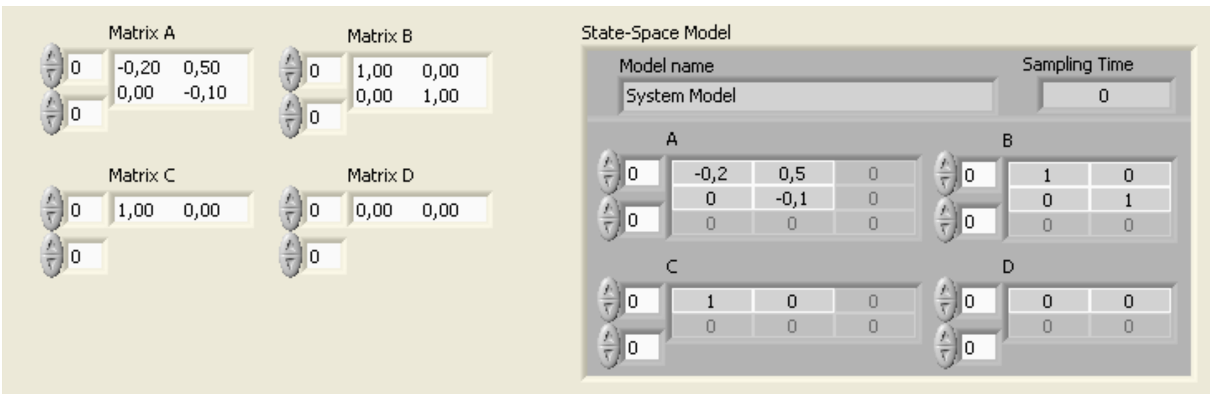
Note! If you specify a discrete State-space model you have to specify the Sampling Time.

LabVIEW Example: Create a State-space model

Block Diagram:



The matrices A, B, C and D may be defined on the Front Panel like this:



[End of Example]

8.2 Eigenvalues

One simple way to find the eigenvalues is to use the Butterworth eigenvalues from the Butterworth polynomial.

Butterwort Polynomial:

The Butterworth Polynomial is defined as:

$$B_n(s) = a_n s^n + \ \dots + a_2 s^2 + a_1 s + 1$$

where $a_0 = 1, a_1, a_2, \dots, a_n$ are the coefficients in the Butterworth Polynomial.

Here we will use a 2.order Butterworth Polynomial, which is defined as:

$$B_2(s) = a_2 s^2 + a_1 s + 1$$

where $a_0 = 1, a_1 = \sqrt{2}T, a_2 = T^2$.

This gives:

$$\boxed{B_2(s) = T^2 s^2 + \sqrt{2}Ts + 1}$$

where the parameter T is used to defined the speed of the response according to:

$$\boxed{T_r \approx nT}$$

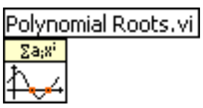
where T_r is defined as the Observer response time where the step response reach 63% of the steady state value of the response.

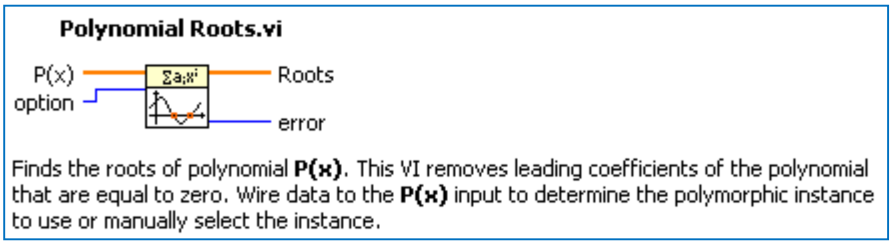
→ So we will use T_r as the tuning parameter for the Observer.

LabVIEW:

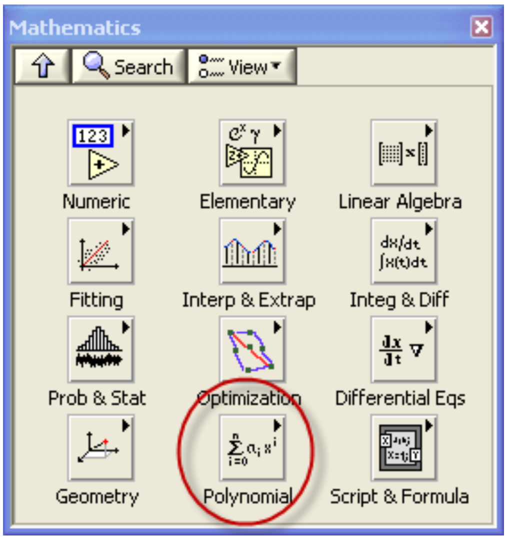
In LabVIEW we can use the "**Polynomial Roots.vi**" to find the roots based on the Butterworth Polynomial

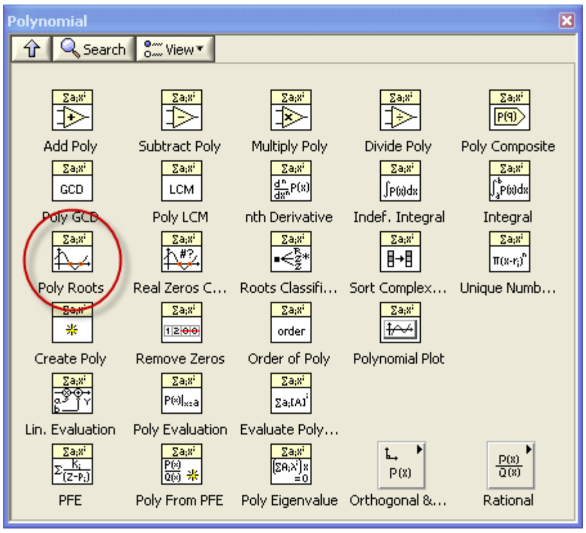
LabVIEW Functions Palette: Mathematics → Polynomial → Polynomial Roots.vi





Below we see the Mathematics and the Polynomial palettes in LabVIEW.





MathScript:

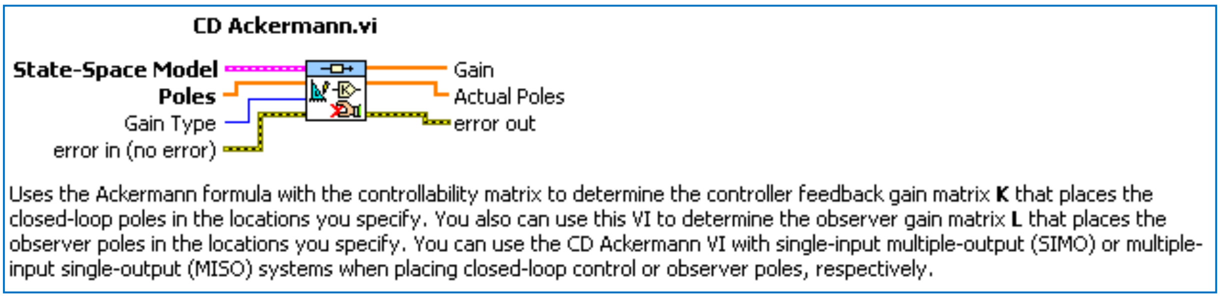
In MathScript we can use the **roots** function in order to find the eigenvalues based on a given polynomial.

8.3 Observer Gain

LabVIEW:

In LabVIEW we can use the "**CD Ackerman.vi**" to find the Observer gain based on some given eigenvalues (found from the Butterwort Polynomial).

LabVIEW Functions Palette: Control Design & Simulation → Control Design → State Feedback Design → CD Ackerman.vi



MathScript:

In MathScript we can use the **acker** function in order to find the Observer gain based on some given eigenvalues (found from the Butterwort Polynomial).

8.4 Observability

A necessary condition for the Observer to work correctly is that the system for which the states are to be estimated, is observable. Therefore, you should check for Observability before applying the Observer.

The Observability matrix is defined as:

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

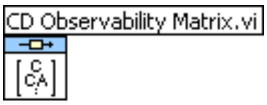
Where n is the system order (number of states in the State-space model).

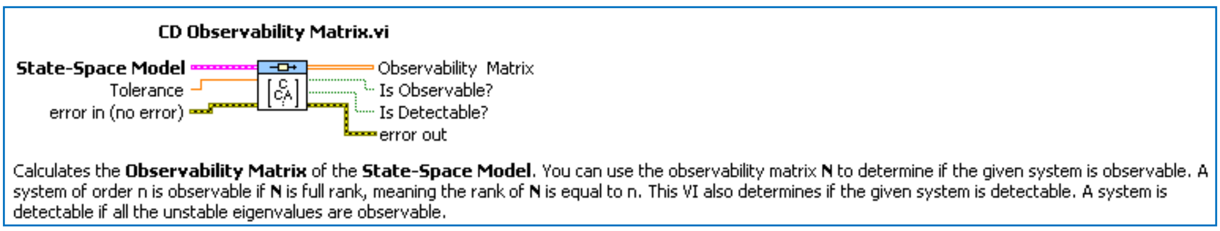
→ A system of order n is observable if O is full rank, meaning the rank of O is equal to n.

LabVIEW:

The LabVIEW Control Design and Simulation Module have a VI (**Observability Matrix.vi**) for finding the Observability matrix and check if a states-pace model is Observable.

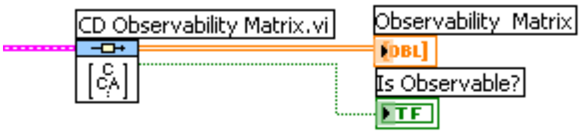
LabVIEW Functions Palette: Control Design & Simulation → Control Design → State-Space Model Analysis → CD Observability Matrix.vi





Note! In LabVIEW N is used as a symbol for the Observability matrix.

LabVIEW Example: Check for Observability



[End of Example]

MathScript:

In MathScript you may use the **obsvmx** function to find the Observability matrix. You may then use the **rank** function in order to find the rank of the Observability matrix.

MathScript Example:

The following MathScript Code check for Observability:

```
% Check for Observability:
O = obsvmx (discretemodel)
r = rank(O)
```

[End of Example]

8.5 Examples

Here we will show implementations of an Observer in LabVIEW and MathScript.

Given the following linear state-space model of a water tank:

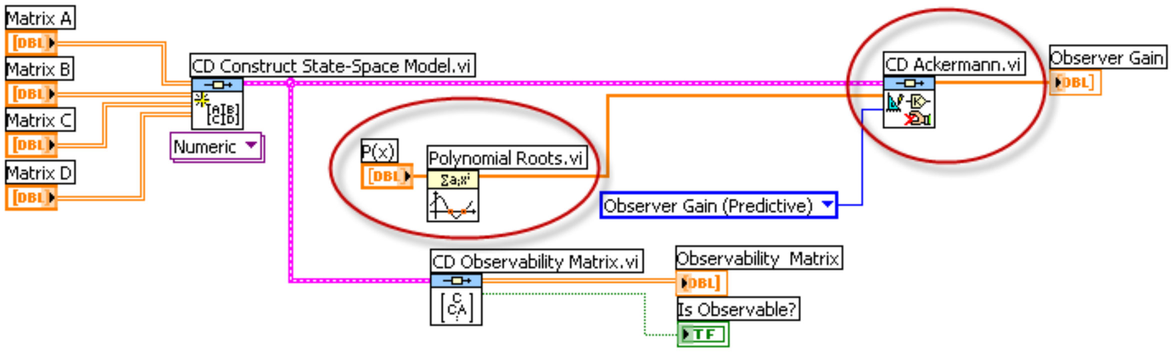
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -10 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0.02 \\ 0 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{[0]}_{D} u$$

Where x_1 is the level in the tank, while x_2 is the outflow of the tank. Only the level x_1 is measured.

LabVIEW Example: Observer Gain

Below we see the Block Diagram in LabVIEW for calculating the Observer gain:



We used the "**Polynomial Roots.vi**" in order to find the poles as specified in the Butterworth Polynomial.

We use a 2.order Butterworth Polynomial:

$$\boxed{B_2(s) = T^2s^2 + \sqrt{2}Ts + 1}$$

where the parameter T is used to defined the speed of the response according to:

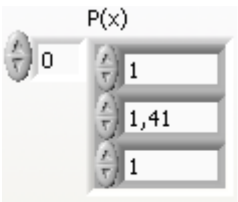
$$\boxed{T_r \approx nT \leftrightarrow T = \frac{T_r}{n}}$$

In the example we set $T_r = 2s$ and $n = 2$ in the example.

This gives:

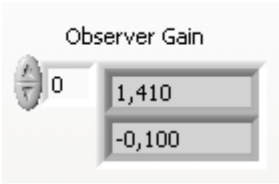
$$B_2(s) = s^2 + 1.41s + 1$$

So the coefficients in the polynomial are as follows:



Then we have used the "CD Akerman.vi" to find the Observer gain.

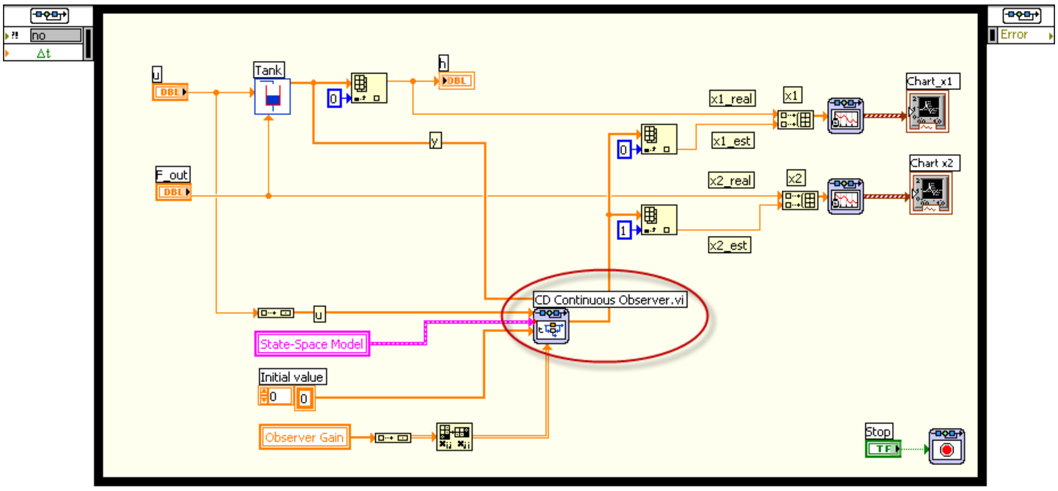
The result becomes:



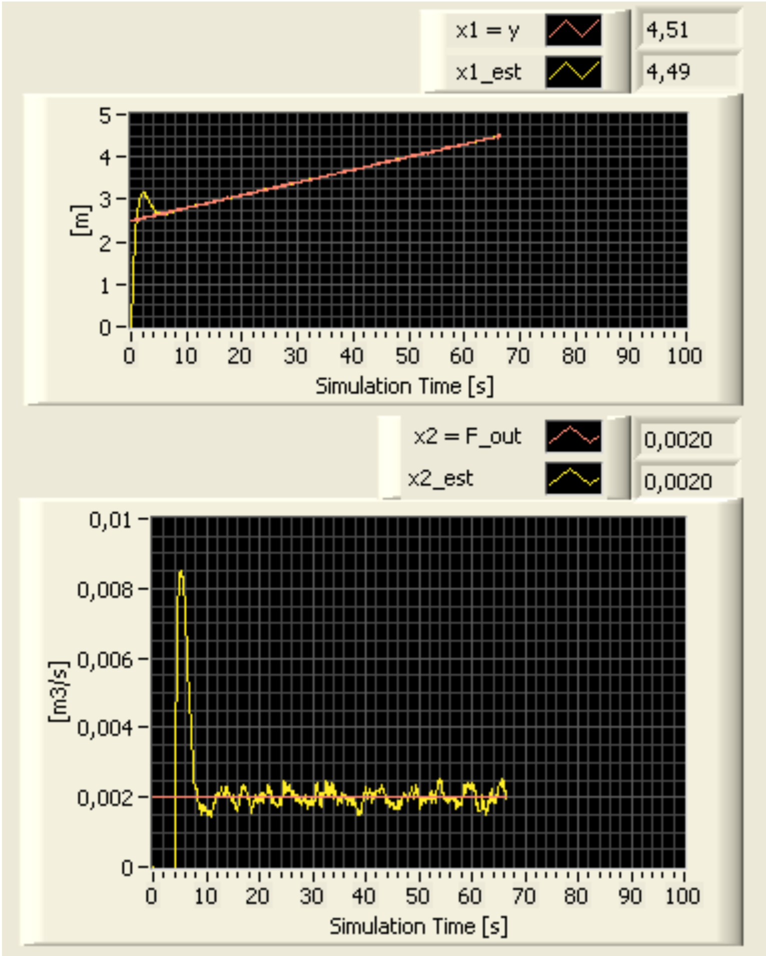
[End of Example]

LabVIEW Example: Observer Estimator

LabVIEW have several built-in Observer functions, e.g., the “CD Continuous Observer.vi” we will use in this example. Below we see the Block Diagram for the Observer:



The result is as follows:



[End of Example]

MathScript Example: Observer Gain

Here we will use MathScript in order to find the Observer gain for the same system as above.

The Code is as follows:

```
% Define the State-space model:
A = [0 -10; 0 0];
B = [0.02; 0];
C = [1 0];
D=[0];
ssmodel = ss(A, B, C,D);
% Check for Observability:
O = obsvmx (ssmodel);
r = rank(O);
%Butterwort Polynomial:
B2=[1, 1.41, 1];
p=roots(B2);
% Find Observer Gain
K = ackermann(ssmodel, p, 'L')
```

The result is:

```
K = 1.41
    -0.1
```

[End of Example]

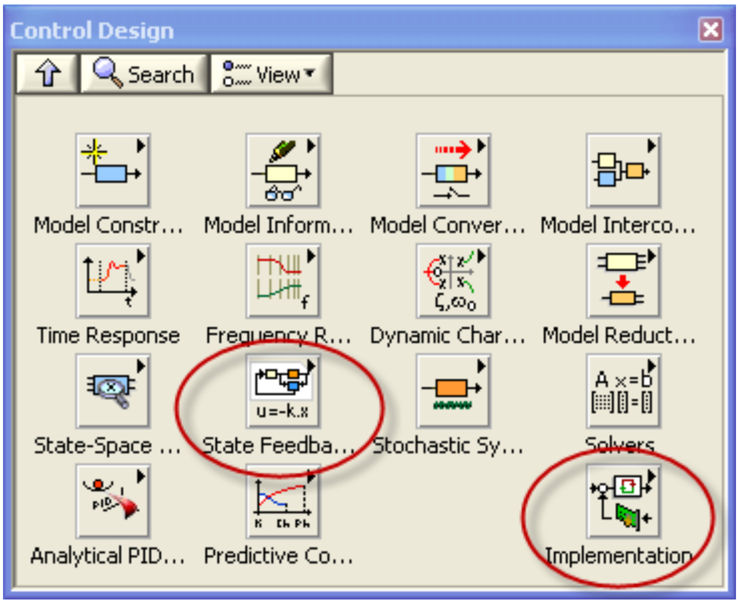
9 Overview of Observer functions

Observers are very similar to Kalman filters. In observers the estimator gain is calculated from specified eigenvalues or poles of the estimator error dynamics (in other words: how fast you want the estimation error to converge to real states).

In LabVIEW there are several VIs and functions used for Observer implementations.

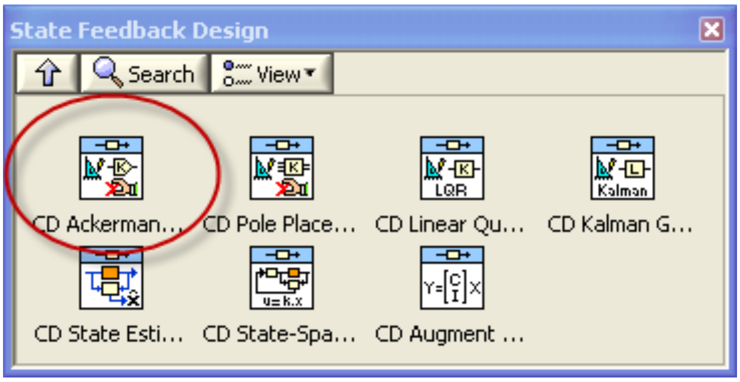
9.1 Control Design palette

In the "**Control Design**" palette we find subpalettes for "**State Feedback Design**" and "**Implementation**":



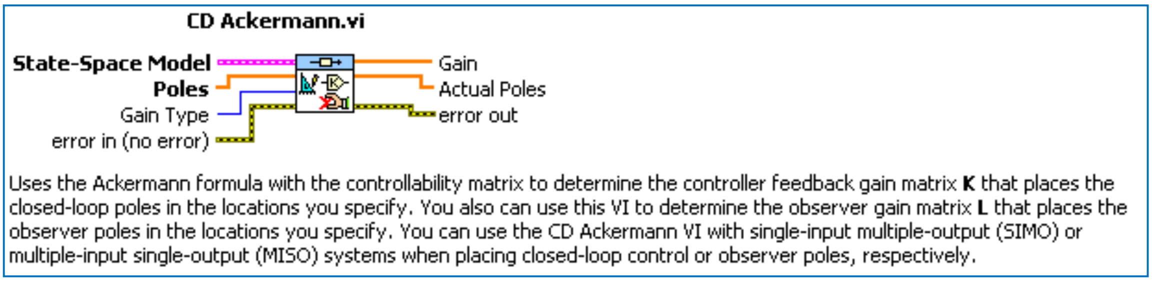
9.1.1 State Feedback Design subpalette

In the "**State Feedback Design**" subpalette we find VIs for calculation the Observer Gain, etc.



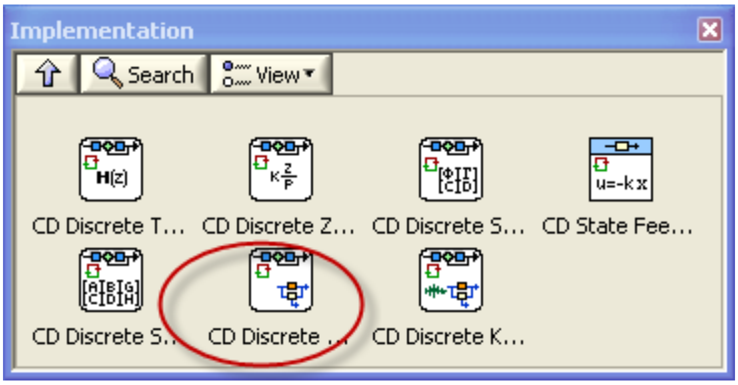
→ Use the State Feedback Design VIs to calculate controller and observer gains for closed-loop state feedback control or to estimate a state-space model. You also can use State Feedback Design VIs to configure and test state-space controllers and state estimators in time domains.

Ackermann VI:



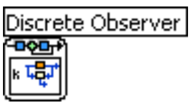
9.1.2 Implementation subpalette

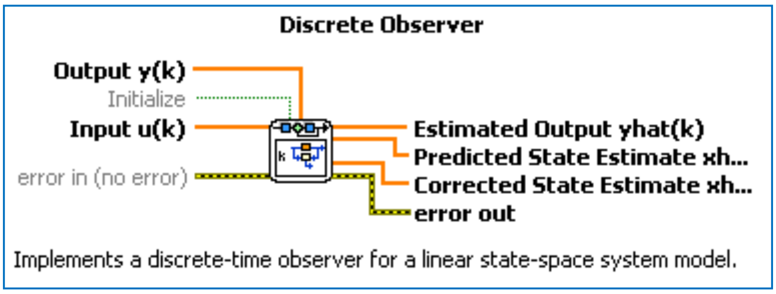
In the "**Implementation**" subpalette we find VIs for implementing a discrete Observer and a discrete Kalman Filter.



→ Use the Implementation VIs and functions to simulate the dynamic response of a discrete system model, deploy a discrete model to a real-time target, implement a discrete Kalman filter, and implement current and predictive observers.

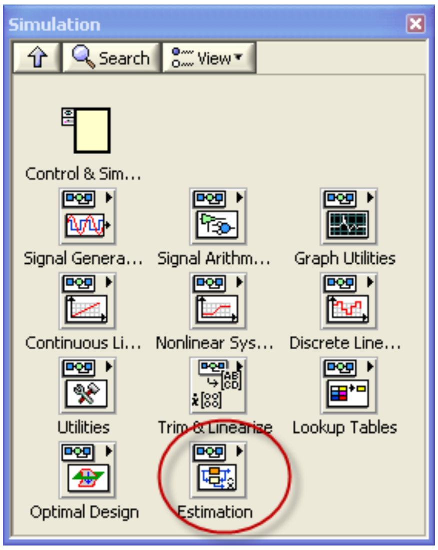
Discrete Observer:





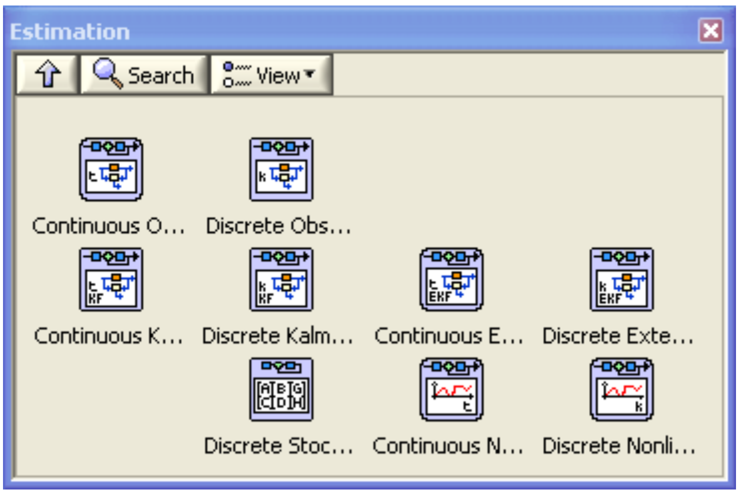
9.2 Simulation palette

In the “**Simulation**” palette we find the “**Estimation**“ sub-palette:



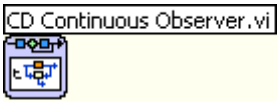
9.2.1 Estimation subpalette

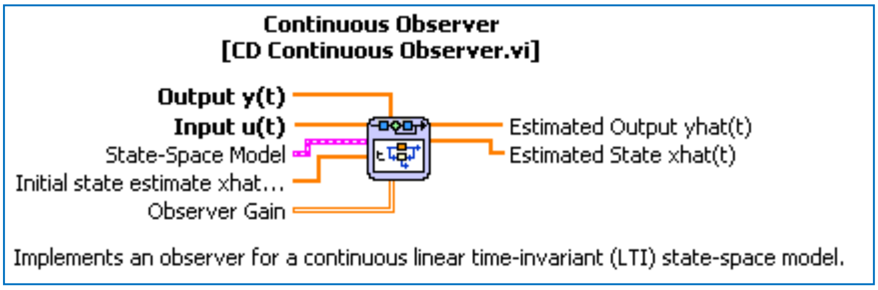
In the “**Estimation**” palette we find VIs for implementing continuous/discrete Observers and Kalman Filter.



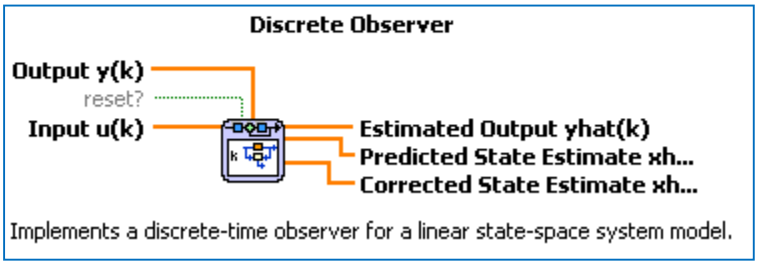
→ Use the Estimation functions to estimate the states of a state-space system. The state-space system can be deterministic or stochastic, continuous or discrete, linear or nonlinear, and completely or partially observable.

Continuous Observer VI:





Discrete Observer VI:



Part III: System Identification

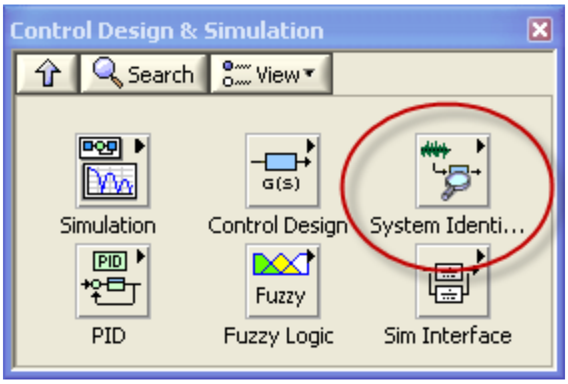
10 System Identification in LabVIEW

The model can be in form of differential equations developed from physical principles or from transfer function models, which can be regarded as "black-box"-models which expresses the input-output property of the system. Some of the parameters of the model can have unknown or uncertain values, for example a heat transfer coefficient in a thermal process or the time-constant in a transfer function model. We can try to estimate such parameters from measurements taken during experiments on the system.

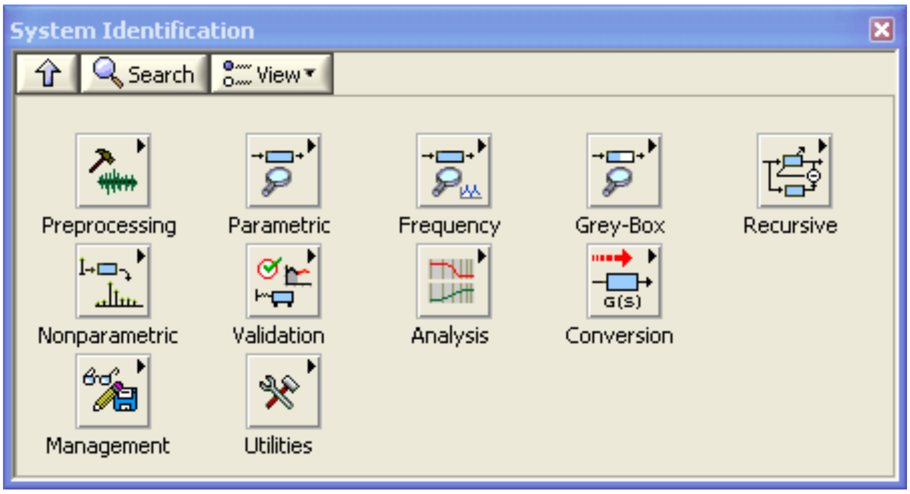
Here we will discuss:

- Parameter Estimation and the Least Square Method (LS)
- Sub-space methods/Black-Box methods
- Polynomial Model Estimation: ARX/ARMAX model Estimation

In LabVIEW we can use the "System Identification Palette".



The "**System Identification**" palette in LabVIEW:



In the next chapters we will use the different functionality available in the System Identification Toolkit.

10.1 Parameter Estimation with Least Square Method (LS)

Parameter Estimation using the Least Square Method (LS) is used to find a model with unknown physical parameters in a mathematical model.

The Least square method can be written as:

$$\boxed{Y = \Phi\theta}$$

Where

$\boldsymbol{\theta}$ **is the unknown parameter vector**

Y is the known measurement vector

Φ is the known regression matrix

The solution for θ may be found as:

$$\theta = \Phi^{-1}Y$$

It can be found that the least square solution for $Y = \Phi Y$ is:

$$\boxed{\theta_{LS} = (\Phi^{\mathrm{T}}\Phi)^{-1}\Phi^{\mathrm{T}}\mathrm{Y}}$$

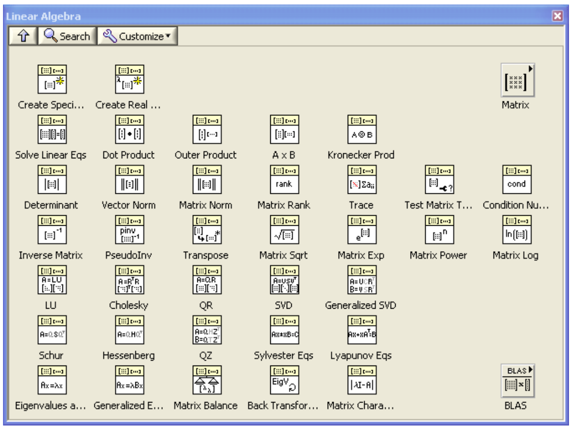
Implementation in MathScript/MATLAB:

```
theta=inv(phi'*phi)* phi'*Y
```

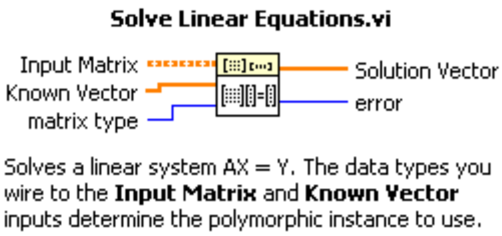
or simply:

```
theta=phi\Y
```

In LabVIEW we can use the blocks ("AxB.vi", "Transpose Matrix.vi", "Inverse Matrix.vi") in the "**Linear Algebra**" (located in the Mathematics palette) palette in order to fin the Least Square solution:



We can also use the "**Solve Linear Equations.vi**":



Example:

Given the following model:

$$y(u) = au + b$$

The following values are found from experiments:

$$y(1) = 0.8$$

$$y(2) = 3.0$$

$$y(3) = 4.0$$

We will find the unknowns a and b using the Least Square (LS) method in **MathScript/LabVIEW**.

We have that:

$$Y = \Phi\theta$$

Where

θ is the unknown parameter vector

Y is the known measurement vector

Φ is the known regression matrix

The solution for θ may be found as (if Φ is a quadratic matrix):

$$\theta = \Phi^{-1}Y$$

It can be found that the least square solution for $Y = \Phi\theta$ is:

$$\boxed{\theta_{LS} = (\Phi^{\mathrm{T}}\Phi)^{-1}\Phi^{\mathrm{T}}\mathrm{Y}}$$

We get:

$$0.8 = a \cdot 1 + b$$

$$3.0 = a \cdot 2 + b$$

$$4.0 = a \cdot 3 + b$$

This becomes:

$$\underbrace{\begin{bmatrix} 0.8 \\ 3.0 \\ 4.0 \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}_{\Phi} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\theta}$$

MathScript:

We define Y and Φ in MathScript and find θ by:

```
phi = [1 1; 2 1; 3 1];
Y = [0.8 3.0 4.0]';

theta = inv(phi'*phi)* phi'*Y

%or simply by
theta=phi\Y
```

The answer becomes:

```
theta =     1.6

           -0.6
```

i.e.:

$$a = 1.6$$

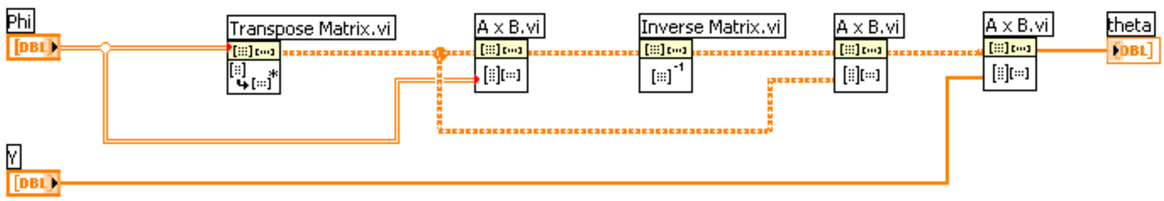
$$b = -0.6$$

This gives:

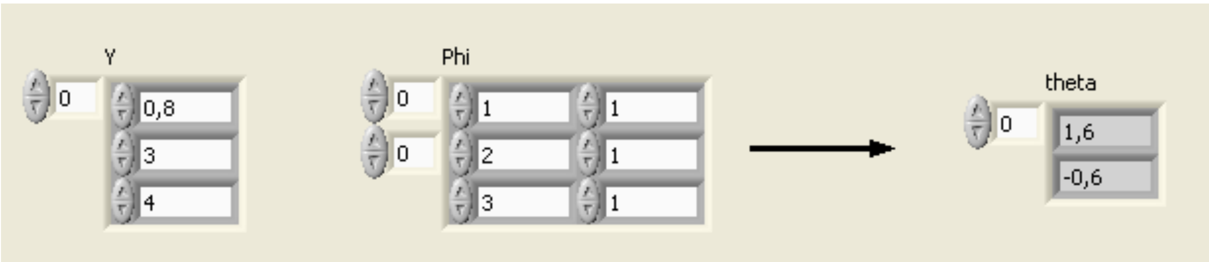
$$\underline{\underline{y(u) = 1.6u - 0.6}}$$

LabVIEW:

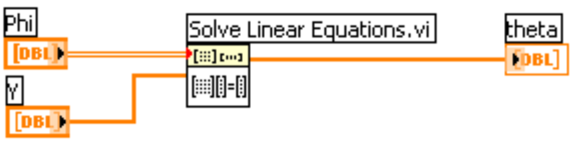
Block Diagram:



Front Panel:



We can also use the “Solve Linear Equations.vi” directly:



[End of Example]

10.2 System Identification using Sub-space methods/Black-Box methods

Sub-space methods/Black-Box methods is used to find a model with non-physical parameters.

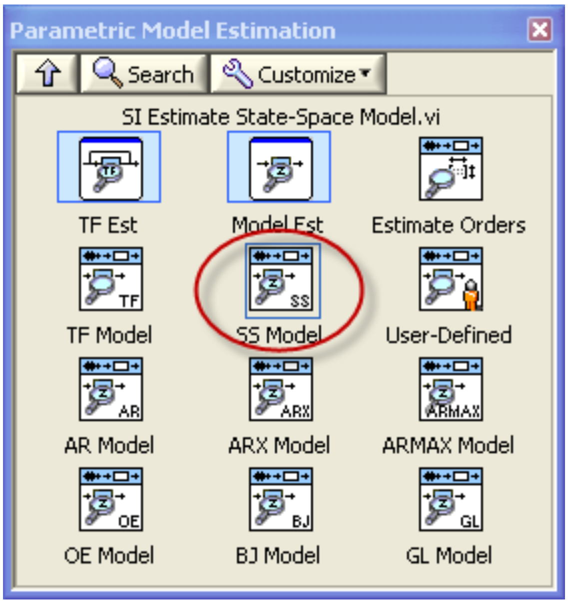
A sub-space methods/Black-Box method estimates a linear discrete State-space model on the form:

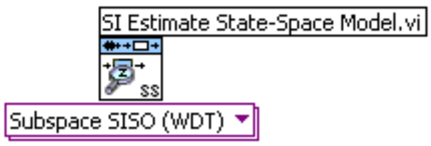
$$x_{k+1} = Ax_k + Bu_k$$

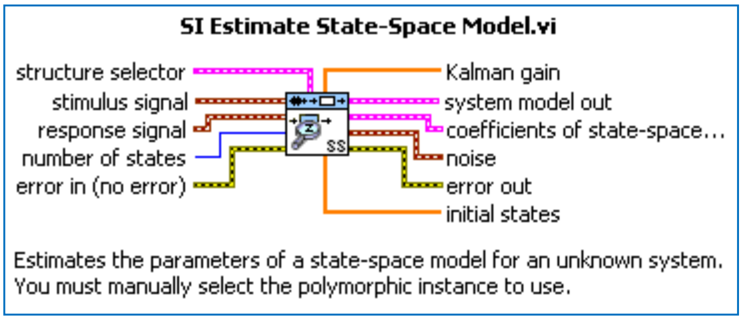
$$y_k = Cx_k + Du_k$$

LabVIEW offers functionality for this. In the “Parametric Model Estimation” palette we find the “**SI Estimate State-Space Model.vi**” which can be used for sub-space identification.

“**Parametric Model Estimation**” palette:



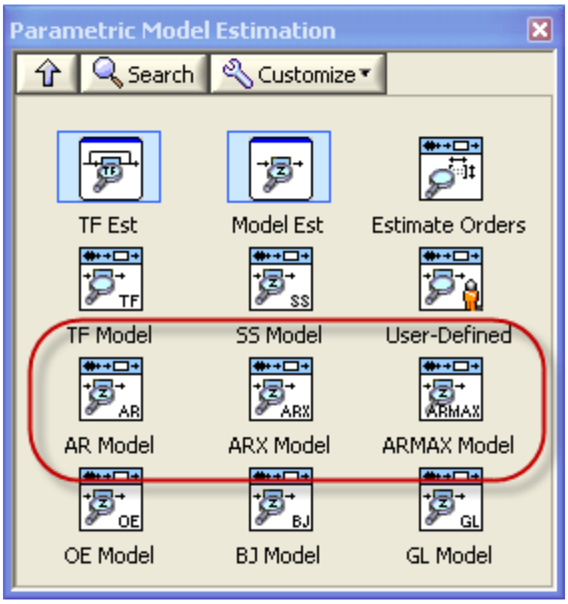




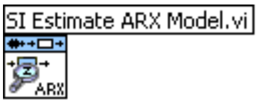
This VI estimates the parameters of a state-space model for an unknown system.

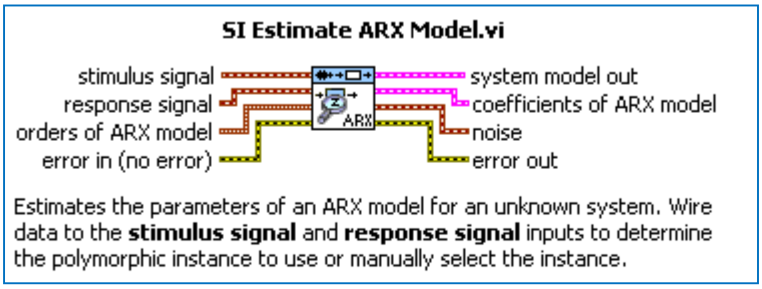
10.3 System Identification using Polynomial Model Estimation: ARX/ARMAX model Estimation

LabVIEW offers VIs for ARX/ARMAX model estimation in the “Parametric Model Estimation” palette.

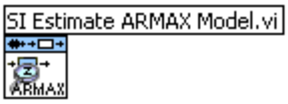


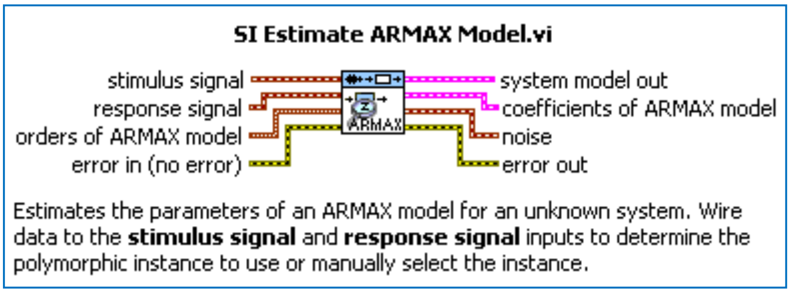
For ARX models we can use “SI Estimate ARX model”:





For ARMAX models we can use “SI Estimate ARMAX model”:





10.4 Generate model Data

In order to find a model we need to generate data based on the real process. The stimulus (exitation) signal and the response signal will then be input to the functions/VIs (algorithms) in LabVIEW that you will use to model your process.

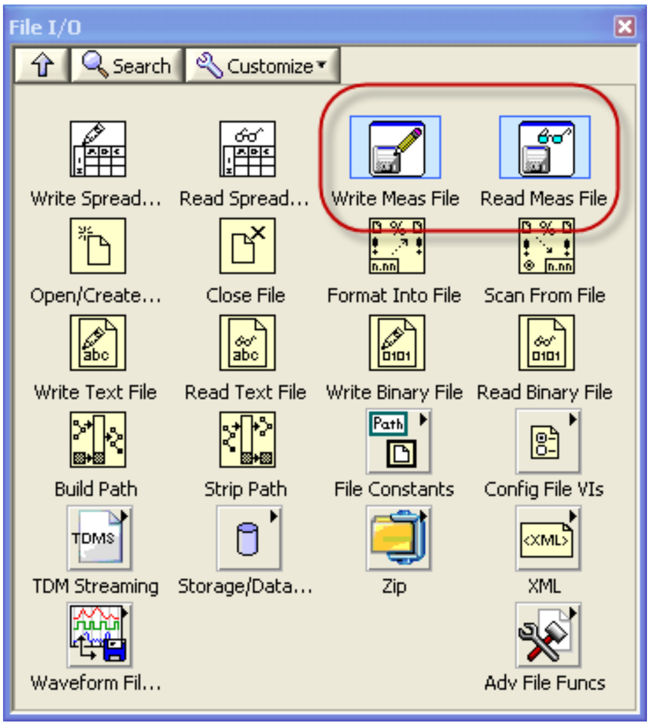
Below we explain how we do this in LabVIEW.

Datalogging:

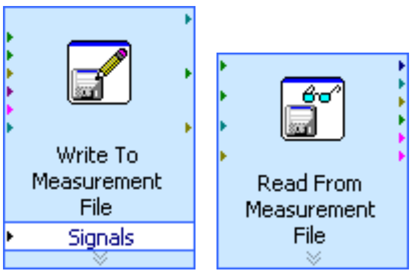
Use LabVIEW for exciting the process and logging signals. Use open-loop experiments (no feedback control system). You can use the Write to Measurement File function on the File I/O palette in LabVIEW for writing data to text files (use the LVM data file format, not the TDMS file format which give binary files).

In the File I/O palette in LabVIEW we have lots of functionality for writing and reading files.

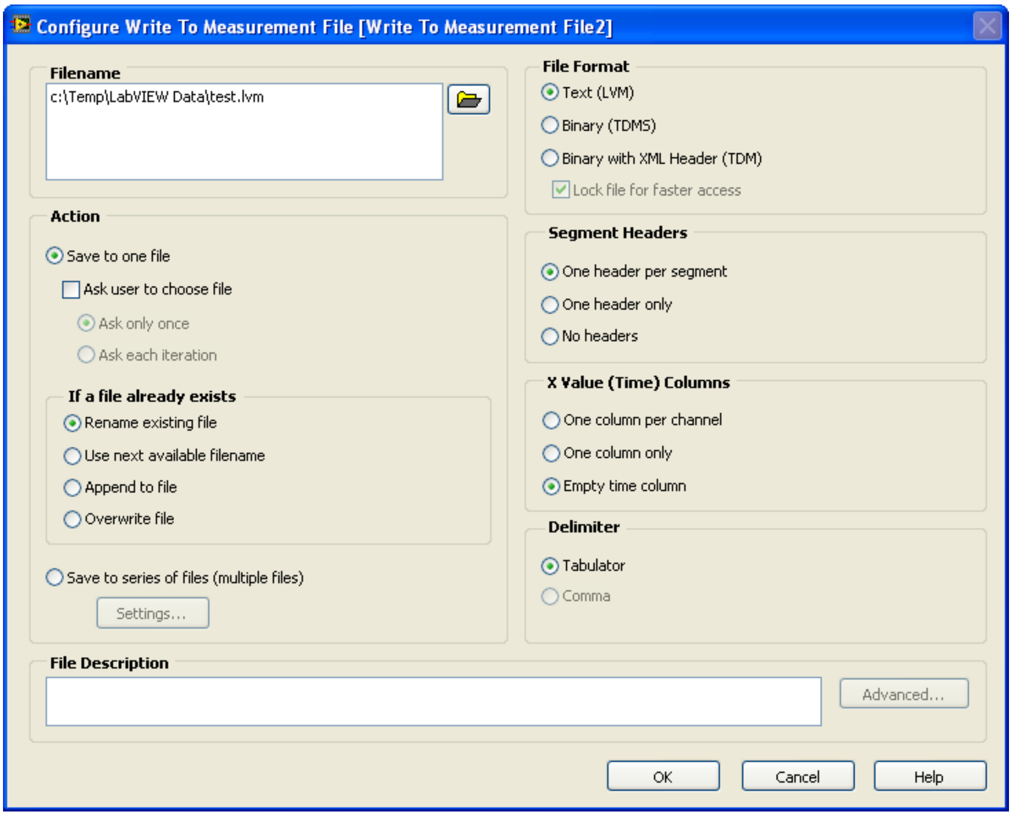
Below we see the "**File I/O**" palette in LabVIEW:



In this Tutorial we will focus on the "**Write To Measurement File**" and "**Read From Measurement File**".



The "Write To Measurement File" and "Read From Measurement File" is so-called "Express VIs". When you drag these VI's to the Block Diagram, a configuration dialog pops-up immediately, like this:



In this configuration dialog you set file name, file type, etc.

Note that these “Express VIs” have no Block Diagram.

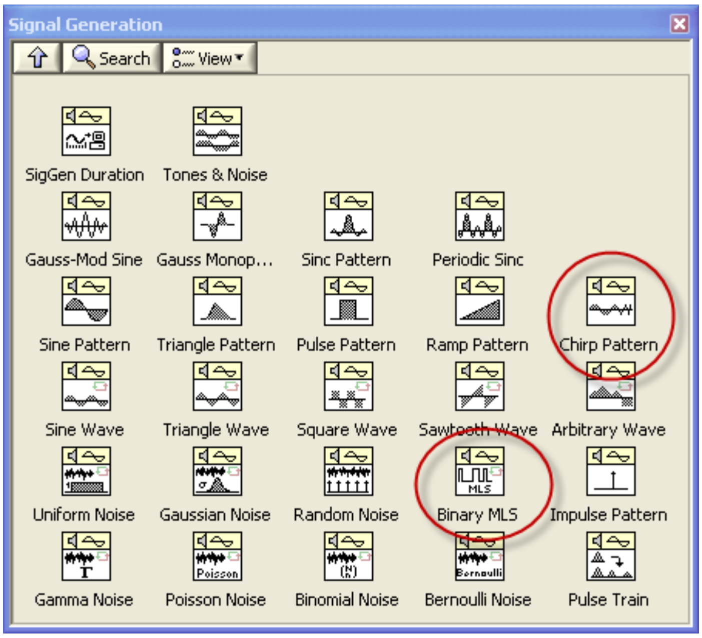
10.4.1 Excitation signals

It is important to have a good excitation signal, you can use different excitation signals, such as:

- A **PRBS signal** (Pseudo Random Binary Signal)
- A **Chirp** Signal
- A **Up-down** signal

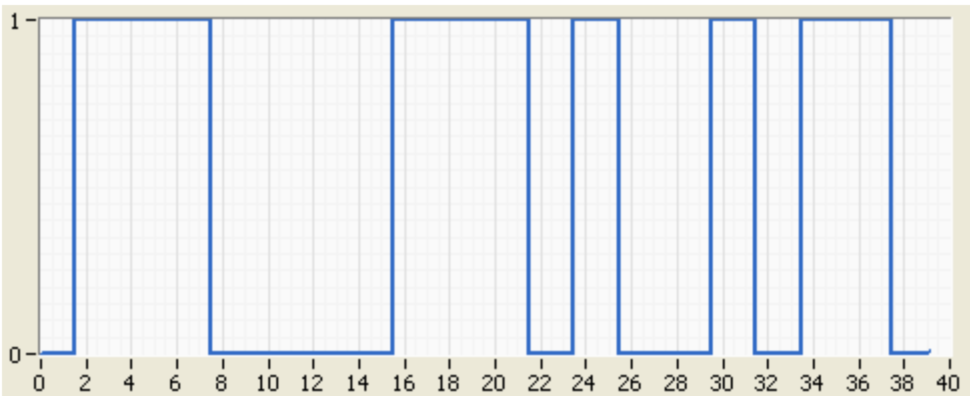
LabVIEW:

In LabVIEW you can use some of the functions in the Signal Generation palette:



LabVIEW Functions Palette: Signal Processing → Signal Generation

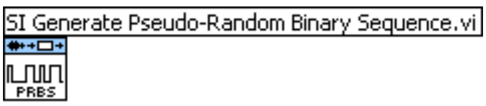
PRBS Signal
A PRBS signal looks like this:

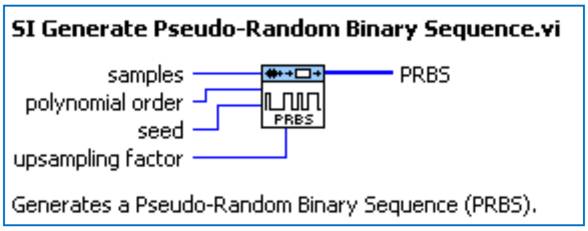


LabVIEW:

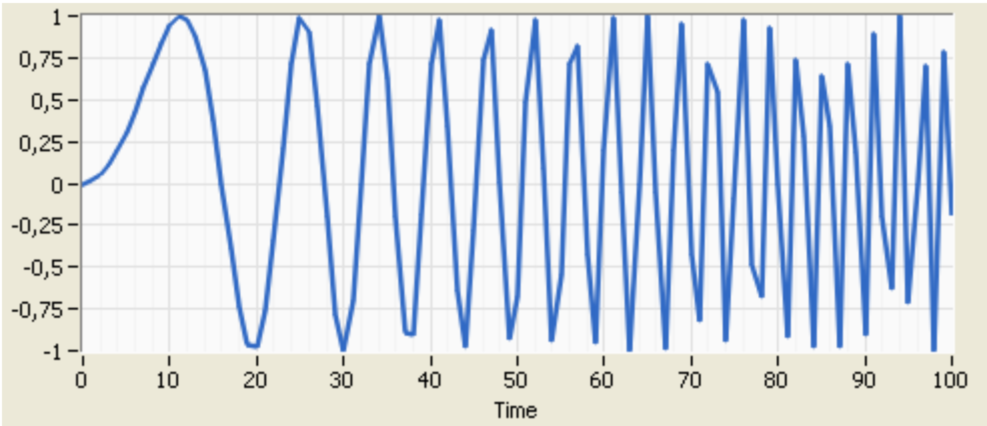
In LabVIEW you can use the "**SI Generate Pseudo-Random Binary Sequence.vi**" function.

LabVIEW Functions Palette: Control Design & Simulation → System identification → Utilities → SI Generate Pseudo-Random Binary Sequence.vi





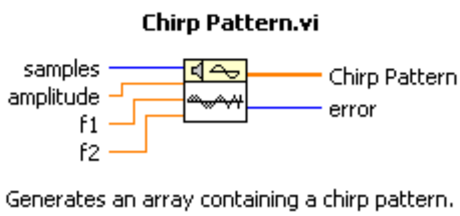
Chirp Signal
A Chirp signal looks like this:



LabVIEW:

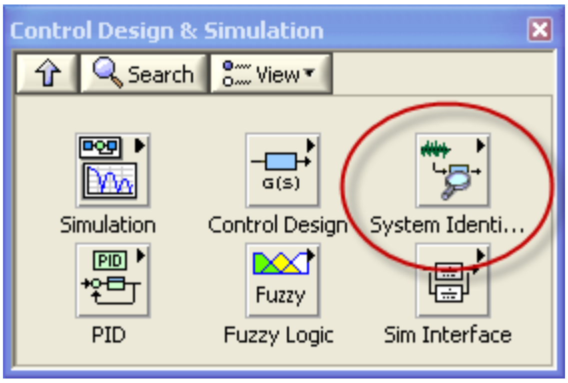
In LabVIEW you can use the "**Chirp Pattern.vi**" function.

LabVIEW Functions Palette: Signal Processing → Signal Generation → Chirp Pattern.vi

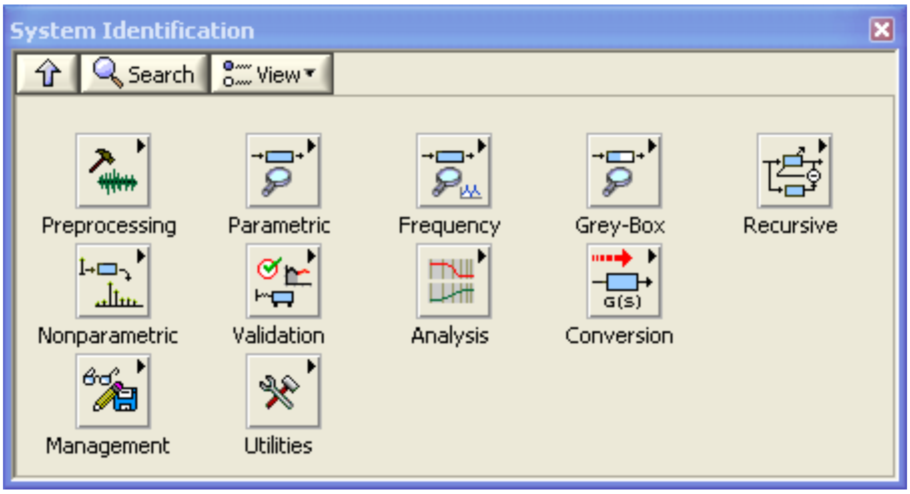


11 Overview of System Identification functions

In LabVIEW we can use the **System Identification Toolkit**.



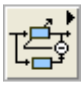
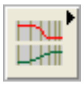
The "**System Identification**" palette in LabVIEW:



→ Use the System Identification VIs to create and estimate mathematical models of dynamic systems. You can use the VIs to estimate accurate models of systems based on observed input-output data.

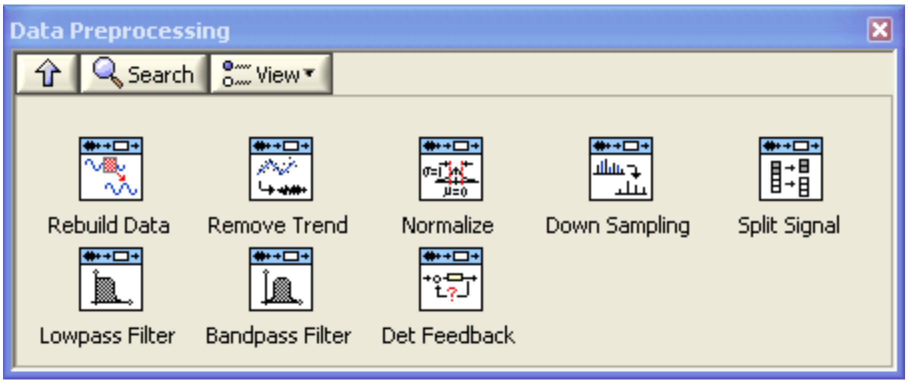
The "**System Identification**" palette in LabVIEW has the following subpalettes:

Icon	Name	Description
 Preprocessing	Data Preprocessing	Use the Data Preprocessing VIs to preprocess the raw data that you acquired from an unknown system.

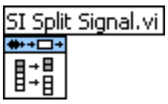
 Parametric	Parametric Model Estimation	Use the Parametric Model Estimation VIs to estimate a parametric mathematical model for an unknown, linear, time-invariant system.
 Frequency	Frequency-Domain Model Estimation	Use the Frequency-Domain Model Estimation VIs to estimate the frequency response function (FRF) and to identify a transfer function (TF) or a state-space (SS) model of an unknown system.
 Grey-Box	Partially Known Model Estimation	Use the Partially Known Model Estimation VIs to create and estimate partially known models for the plant in a system.
 Recursive	Recursive Model Estimation	Use the Recursive Model Estimation VIs to recursively estimate the parametric mathematical model for an unknown system.
 Nonparametric	Nonparametric Model Estimation	Use the Nonparametric Model Estimation VIs to estimate the impulse response or frequency response of an unknown, linear, time-invariant system from an input and corresponding output signal.
 Validation	Model Validation	Use the Model Validation VIs to analyze and validate a system model.
 Analysis	Model Analysis	Use the Model Analysis VIs to perform a Bode, Nyquist, or pole-zero analysis of a system model and to compute the standard deviation of the results.
 Conversion	Model Conversion	Use the Model Conversion VIs to convert models created in the LabVIEW System Identification Toolkit into models you can use with the LabVIEW Control Design and Simulation Module. You can convert an AR, ARX, ARMAX, output-error, Box-Jenkins, general-linear, or state-space model into a transfer function, zero-pole-gain, or state-space model. You also can convert a

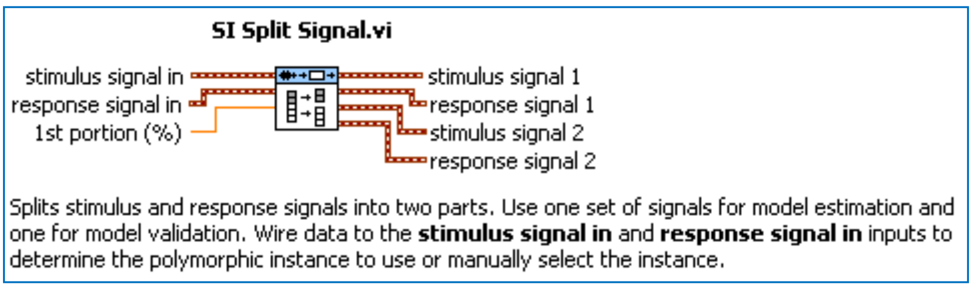
		continuous model to a discrete model or convert a discrete model to a continuous model.
 Managemen t	Model Management	Use the Model Management VIs to access information about the system model. Model information includes properties such as the system type, sampling rate, system dimensions, noise covariance, and so on.
 Utilities	Utilities	Use the Utilities VIs to perform miscellaneous tasks on data or the system model, including producing data samples, displaying model equations, merging models, and so on.

The “**Data Preprocessing**” palette in LabVIEW:

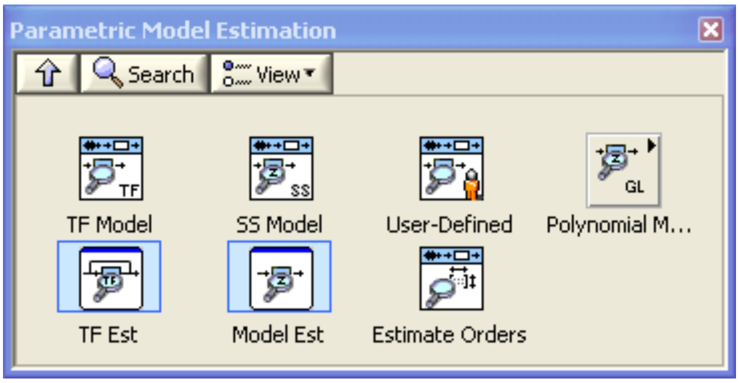


Some important functions in the “Data Preprocessing” palette are:

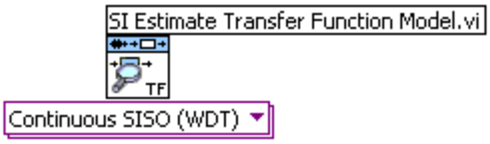


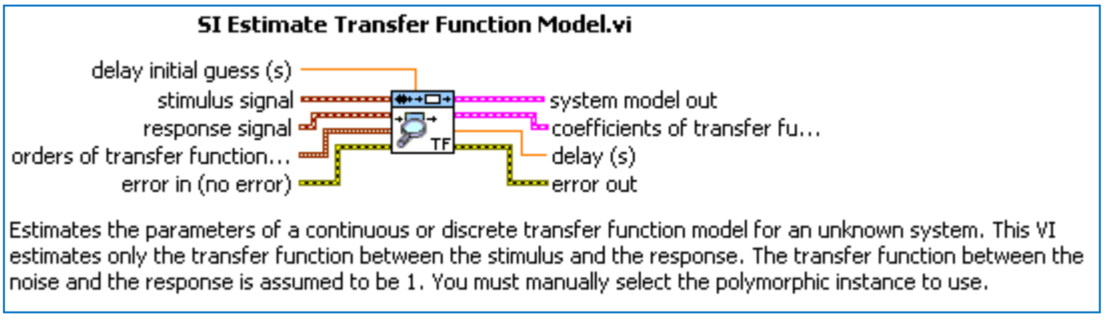


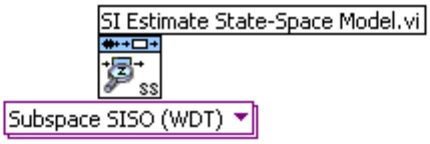
The “**Parametric Model Estimation**” palette in LabVIEW:

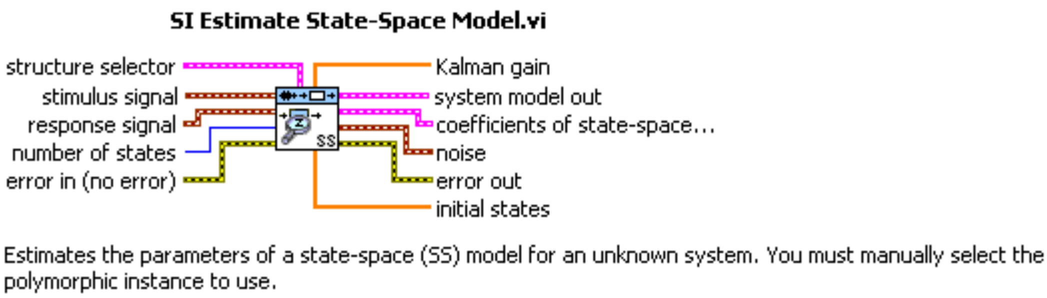


Some important functions in the “Parametric Model Estimation” palette are:

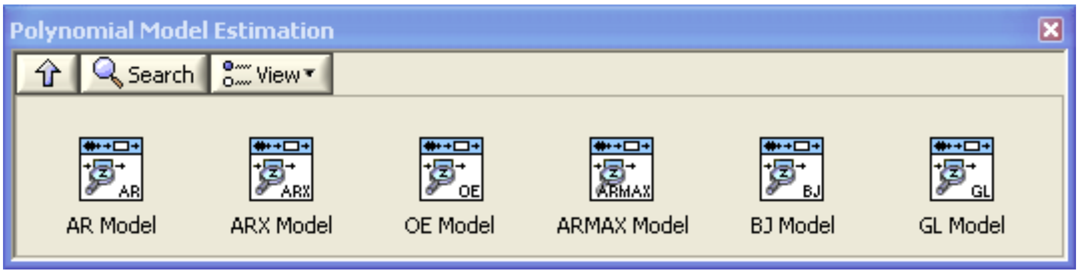






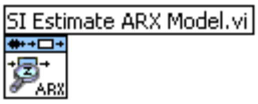


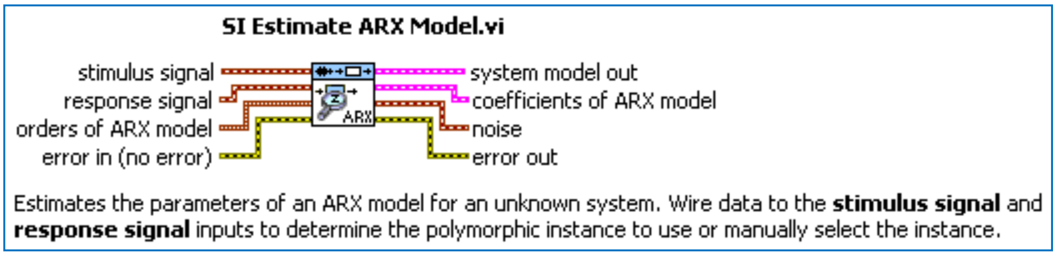
The “Parametric Model Estimation” palette in LabVIEW has subpalette for “**Polynomial Model Estimation**”:

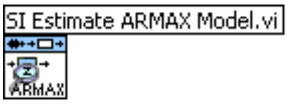


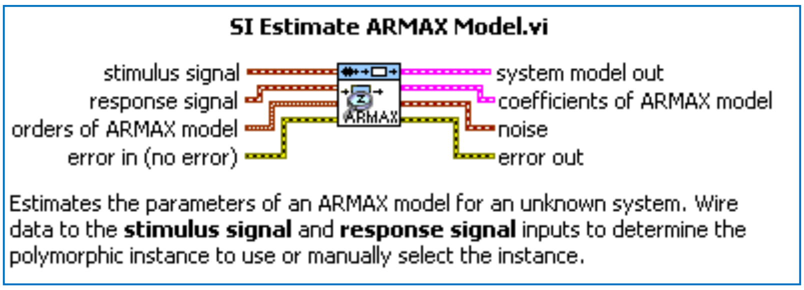
→ Use the Polynomial Model Estimation VIs to estimate an AR, ARX,ARMAX, Box-Jenkins, or output-error model for an unknown, linear, time-invariant system.

Some important functions in the “Polynomial Model Estimation” palette are:









12 System Identification Example

We want to identify the model of a given system.

We have found the model to be:

$$\dot{x} = -\frac{1}{T}x + Ku(t-\tau)$$

where

T is the time constant

K is the system gain, e.g. pump gain

τ is the time-delay

→ We want to find the model parameters T, K, τ using the Least Square method. We will use LabVIEW and MathScript.

Set the system on the form $y = \varphi\theta$

Solutions:

We get:

$$\underbrace{\dot{x}}_{y} = \underbrace{[x \quad u(t-\tau)]}_{\varphi}\underbrace{\begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}}_{\theta}$$

i.e.

$$\theta = \begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}$$

In order to find θ using the Least Square method we need to log input and output data. This means we need to discretize the system.

We use a simple Euler forward method:

$$\dot{x} \approx \frac{x_{k+1} - x_k}{T_s}$$

T_s is the sampling time.

This gives:

$$\underbrace{\frac{x_{k+1} - x_k}{T_s}}_{y} = \underbrace{\begin{bmatrix} x_k & u_{k-\frac{\tau}{T_s}} \end{bmatrix}}_{\varphi} \underbrace{\begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}}_{\theta}$$

Let's assume $\tau = 3s$ (which can be found from a simple step response on the real system), we then get (with sampling time $T_s = 0.1$):

$$\underbrace{\frac{x_{k+1} - x_k}{T_s}}_{y} = \underbrace{\begin{bmatrix} x_k & u_{k-30} \end{bmatrix}}_{\varphi} \underbrace{\begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}}_{\theta}$$

Note! In 3 seconds we log 30 points with data using sampling time $T_s = 0.1$!!!

2 Given the following logging data (the data is just for illustration and not realistic):

k	u	y
1	0.9	3
2	1.0	4
3	1.1	5
4	1.2	6
5	1.3	7
6	1.4	8
7	1.5	9

We use the following sampling time: $\boldsymbol{T_s = 1s}$

From a simple step response, we have found the time-delay to be: $\tau = 3s$.

→ Set the system on the form $Y = \Phi\theta$

Solutions:

With time-delay $\tau = 3s$ and $T_s = 1s$ we get:

$$\underbrace{\frac{x_{k+1} - x_k}{T_s}}_{y} = \underbrace{[x_k \quad u_{k-3}]}_{\varphi} \underbrace{\begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}}_{\theta}$$

Using the given data set we can set on the form $Y = \Phi\theta$:

$$\begin{bmatrix} 7-6 \\ 8-7 \\ 9-8 \end{bmatrix} = \begin{bmatrix} 6 & 0.9 \\ 7 & 1.0 \\ 8 & 1.1 \end{bmatrix} \begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}$$

i.e.:

$$\underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} 6 & 0.9 \\ 7 & 1.0 \\ 8 & 1.1 \end{bmatrix}}_{\Phi} \underbrace{\begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}}_{\theta}$$

Note! We need to make sure the dimensions are correct.

→ We find the model parameters (θ) using MathScript

MathScript gives:

```
clear, clc

Y = [1, 1, 1]';
phi = [6, 0.9; 7, 1.0; 8, 1.1];


theta = phi\Y

%or
theta = inv(phi'*phi)*phi'*Y

T = -1/theta(1)
K = theta(2)
```

MathScript responds with the following answers:

```
theta =

          -0.3333

           3.3333

T =

           3

K =

           3.3333
```

i.e., the model parameters become:

$$T = 3, \qquad K = \frac{10}{3}, \qquad \tau = 3$$

Which gives the following modell:

$$\dot{x} = -\frac{1}{T}x + Ku(t - \tau)$$

With values:

$$\dot{x} = -\frac{1}{3}x + \frac{10}{3}(t - 3)$$

3 Implement the model in LabVIEW. Use $T = 5,\ K = 2,\ \tau = 3$ and simulate the system. Plot the step response for the system.

Model:

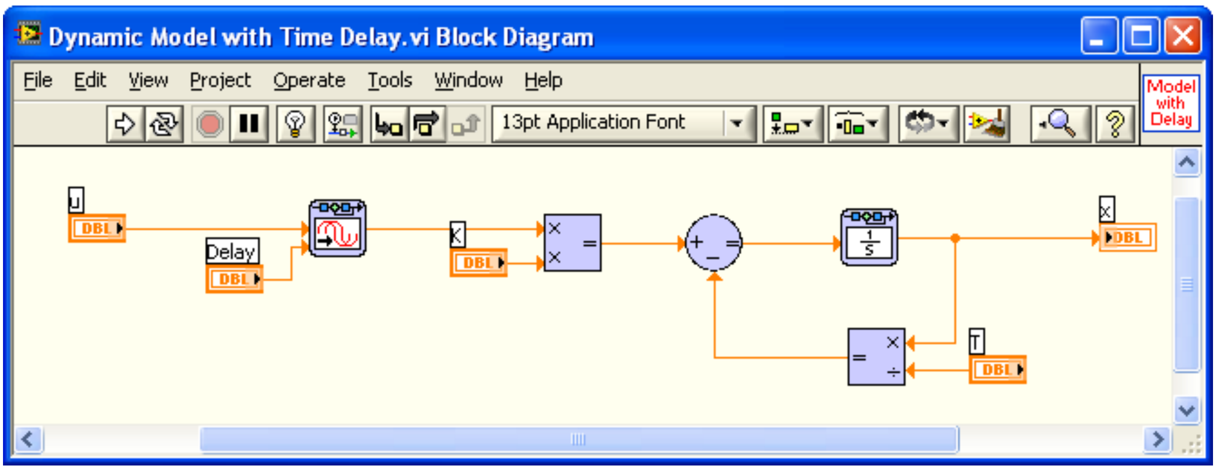
$$\dot{x} = -\frac{1}{T}x + Ku(t - \tau)$$

Where $T = 5,\ K = 2,\ \tau = 3$

Solutions:

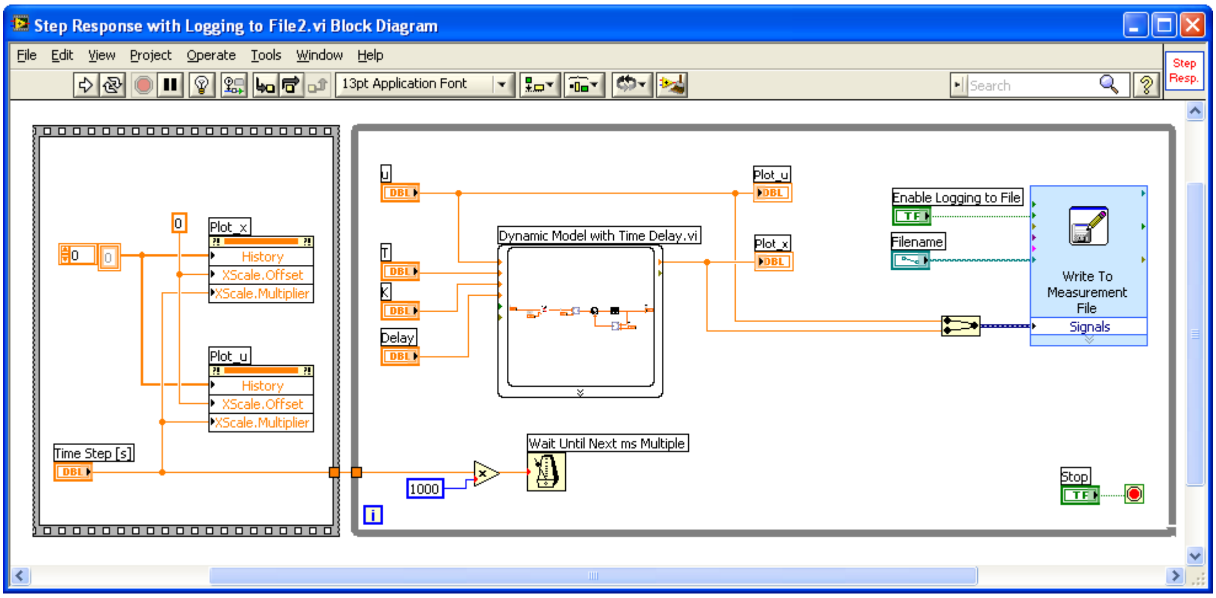
We implement the model using a "Simulation Subsystem" and use the available blocks in the Control and Design Module.

The model may be implemented as follows:



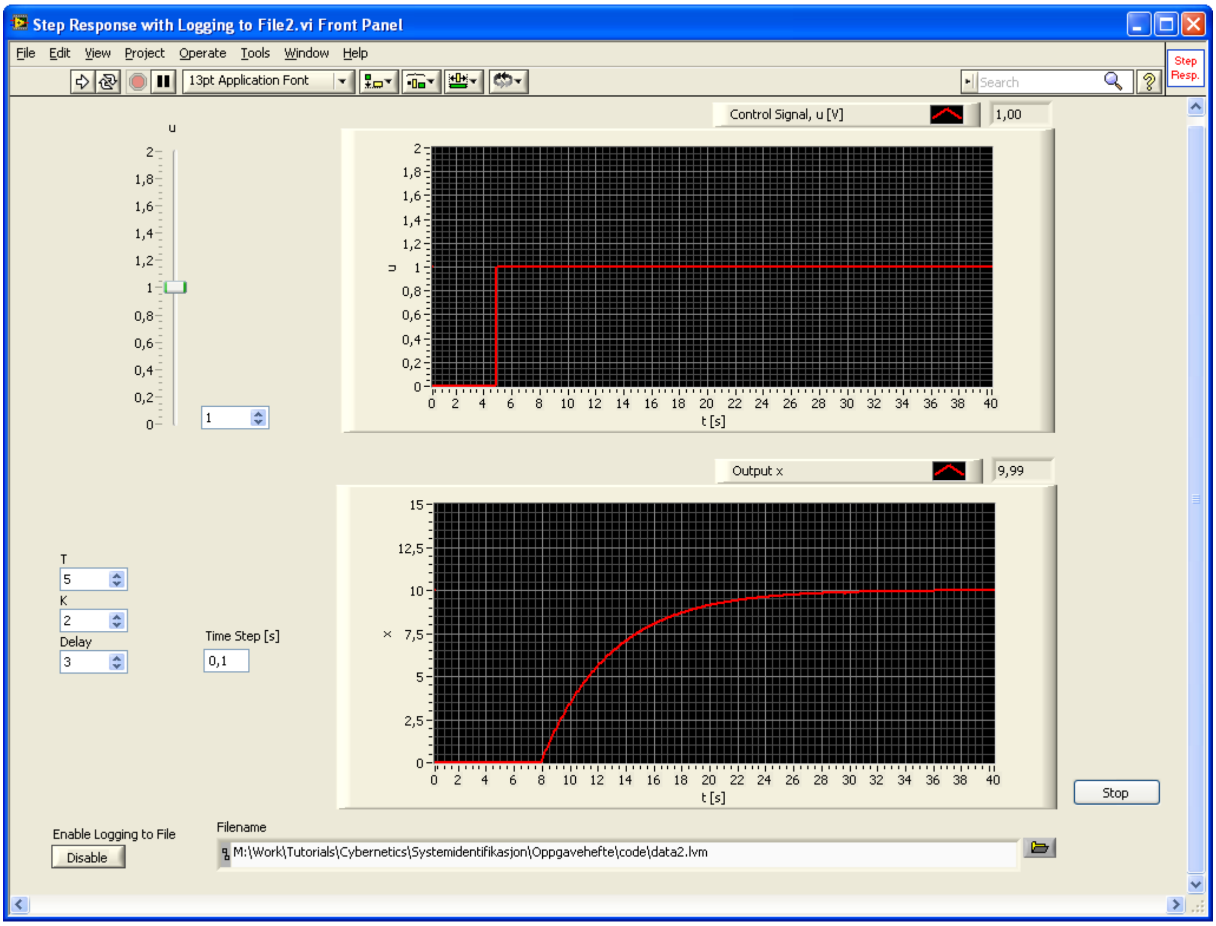
We simulate the system using the following program:

Block Diagram:



Front Panel:

We do a simple step response:



4 Find the transfer function for the system:

$$H(s) = \frac{x(s)}{u(s)}$$

Model:

$$\dot{x} = -\frac{1}{T}x + Ku(t - \tau)$$

Plot the step response for the transfer function in MathScript. Compare and discuss the results from previous task.

Solutions:

We use the differential equation:

$$\dot{x} = -\frac{1}{T}x + Ku(t - \tau)$$

Laplace transformation gives:

$$sx(s) = -\frac{1}{T}x(s) + Ku(s)e^{-\tau s}$$

Note! We use the following Laplace transformation:

$$\boxed{F(s)e^{-\tau s} \Leftrightarrow f(t-\tau)}$$

$$\boxed{sF(s) \ \Leftrightarrow \dot{f}(t)}$$

Then we get:

$$sx(s) + \frac{1}{T}x(s) = Ku(s)e^{-\tau s}$$

and:

$$x(s)\left(s + \frac{1}{T}\right) = Ku(s)e^{-\tau s}$$

and:

$$\frac{x(s)}{u(s)} = \frac{K}{s + \frac{1}{T}}e^{-\tau s}$$

Finally:

$$H(s) = \frac{x(s)}{u(s)} = \frac{KT}{Ts + 1}e^{-\tau s} = \frac{K_{tot}}{Ts + 1}e^{-\tau s}$$

With values ($T = 5,\ K = 2,\ \tau = 3$):

$$H(s) = \frac{x(s)}{u(s)} = \frac{10}{5s + 1}e^{-3s}$$

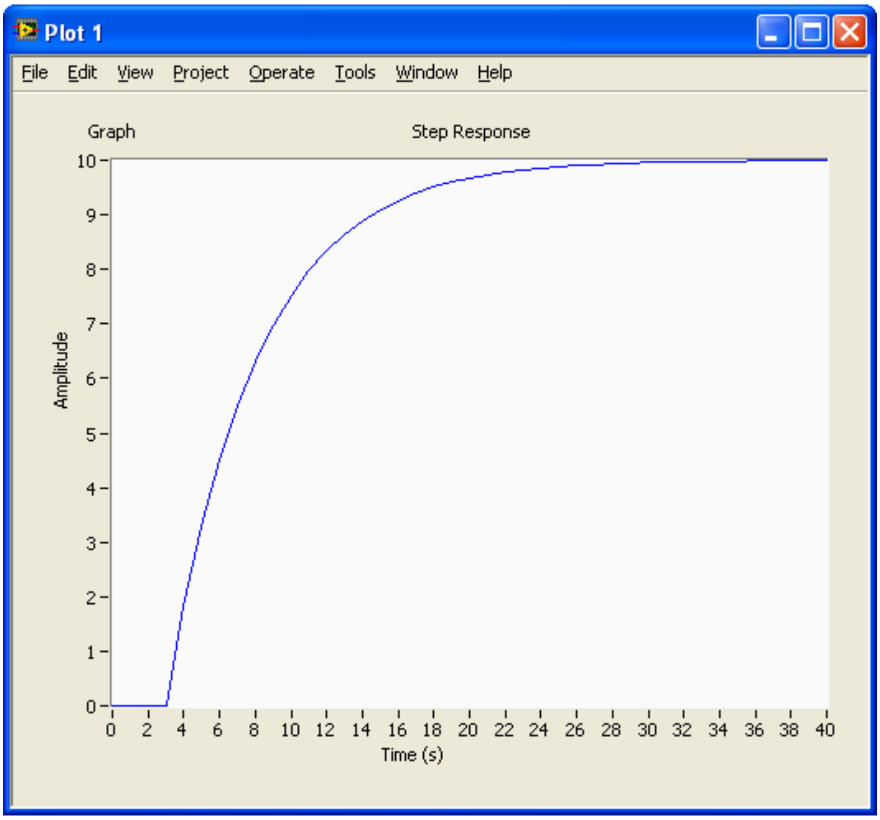
MathScript:

We implement the transfer function in MathScript and perform a step response. We use the *step()* function.

```
clear, clc

s=tf('s');
K=2;
T=5;

H1=tf(K*T/(T*s+1));

delay=3;
H2=set(H1,'inputdelay',delay);
step(H2)
```

```
% You may also use:
figure(2)
H = sys_order1(K*T, T, delay)
step(H)
```

The step response becomes:



→ We see that the step response is the same as in the previous task.

5 Log input and output data based on the model.

Model:

$$\dot{x} = -\frac{1}{T}x + Ku(t-\tau)$$

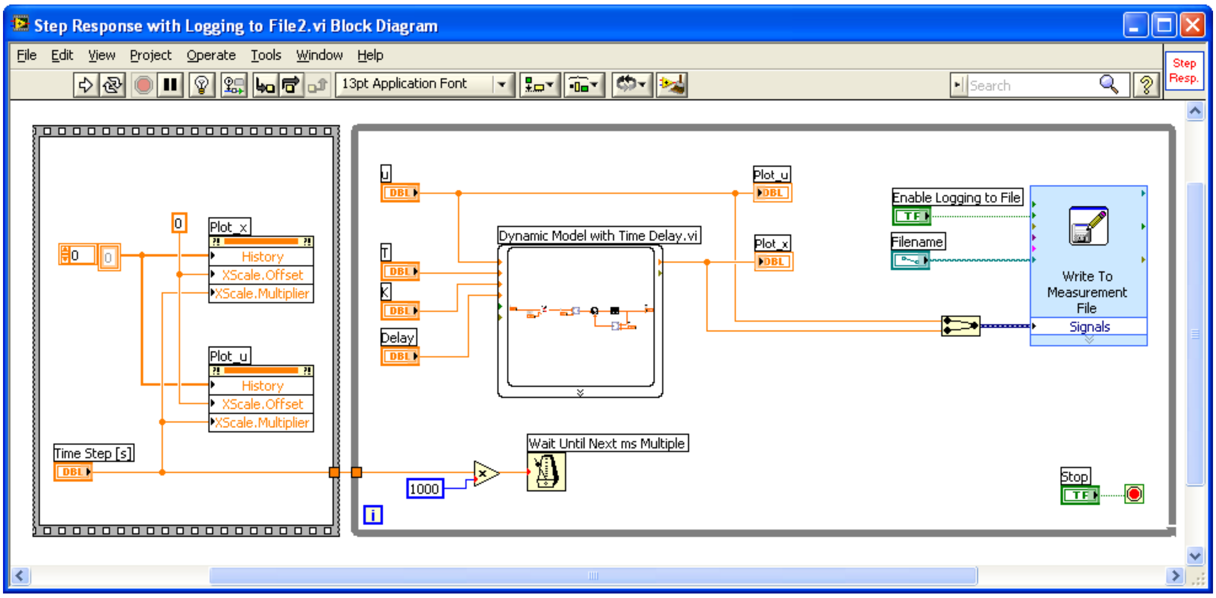
Where $T = 5,\ K = 2,\ \tau = 3$

Solutions:

We save the data using "Write To Measurement File" in LabVIEW.

Based on a simple step response we can find the time-delay τ.

We use the same application as in a previous task:



6 Find the model parameters (T og K) using Least Square in LabVIEW based on the logged data.

Note! The answers should be $T \approx 5$ and $K \approx 2$.

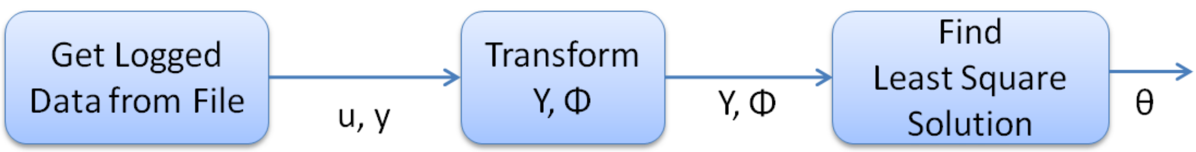
Model:

$$\dot{x} = -\frac{1}{T}x + Ku(t - \tau)$$

Where $T = 5,\ K = 2,\ \tau = 3$

Solutions:

It is a good idea to split your program into different logical parts using, e.g., SubVIs in LabVIEW.



The different parts/steps could, e.g., be:

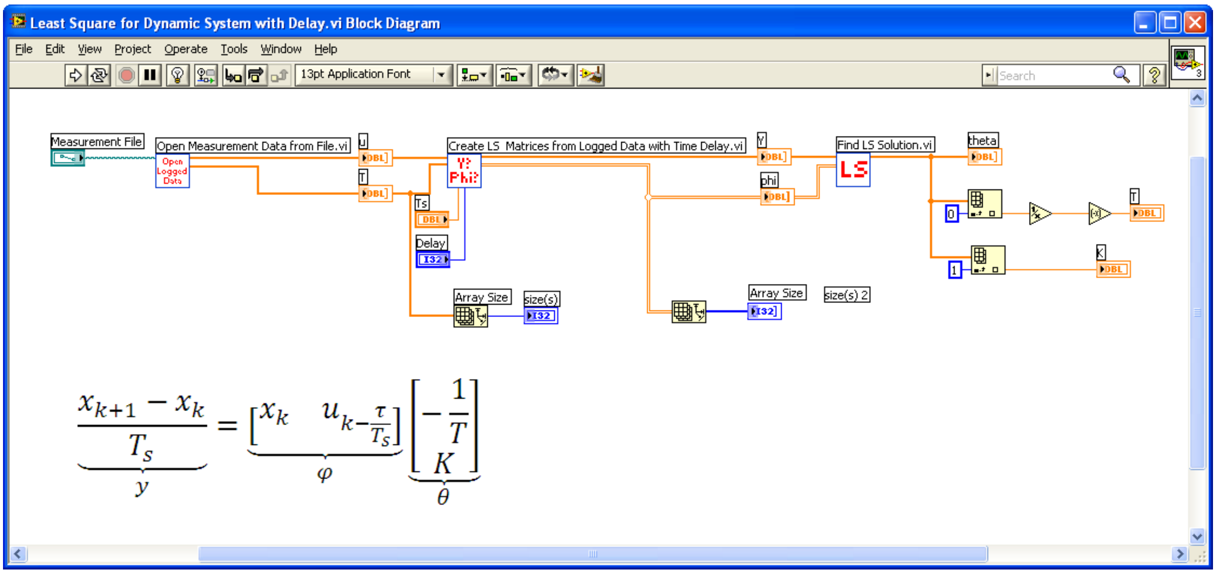
1. Get Logged Data from File
   a. Input: File Name
   b. Outputs: u and y (T_{out})
2. Transform the data and stack data into Y and Φ

   a. Inputs: u and y (T_{out})
   b. Outputs: Y and Φ

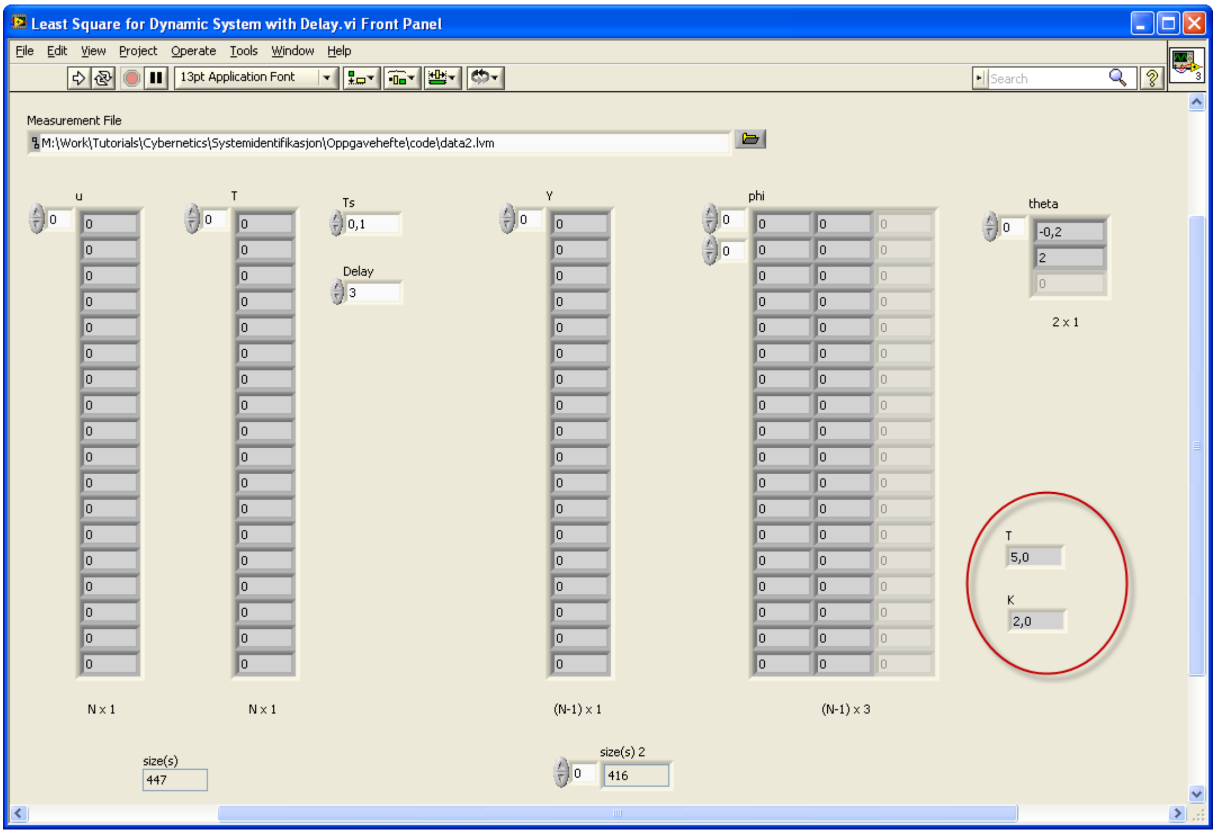
3. Find the Least Square solution $\theta_{LS} = (\Phi^T\Phi)^{-1}\Phi^T Y$
   a. Input: Y and Φ
   b. Output: θ (θ_t, K_h)

LabVIEW code:

Block Diagram:



Front Panel:



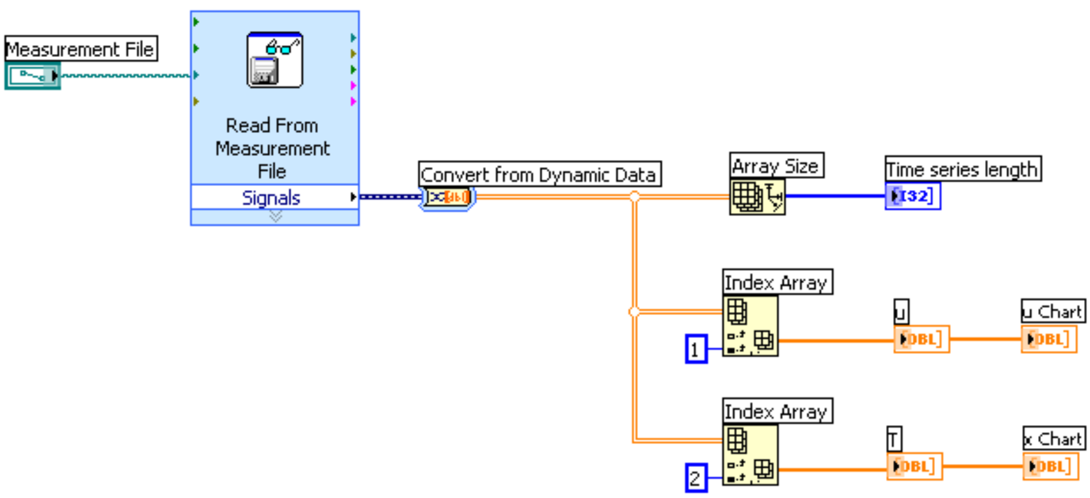
→ As you see the result is $T = 5$ and $K = 2$ (as expected).

The different SubVI's do the following:

<u>1. "Open Measurement Data from File.vi"</u>

This SubVI opens the logged data from file (created in a previous task).

Block Diagram:

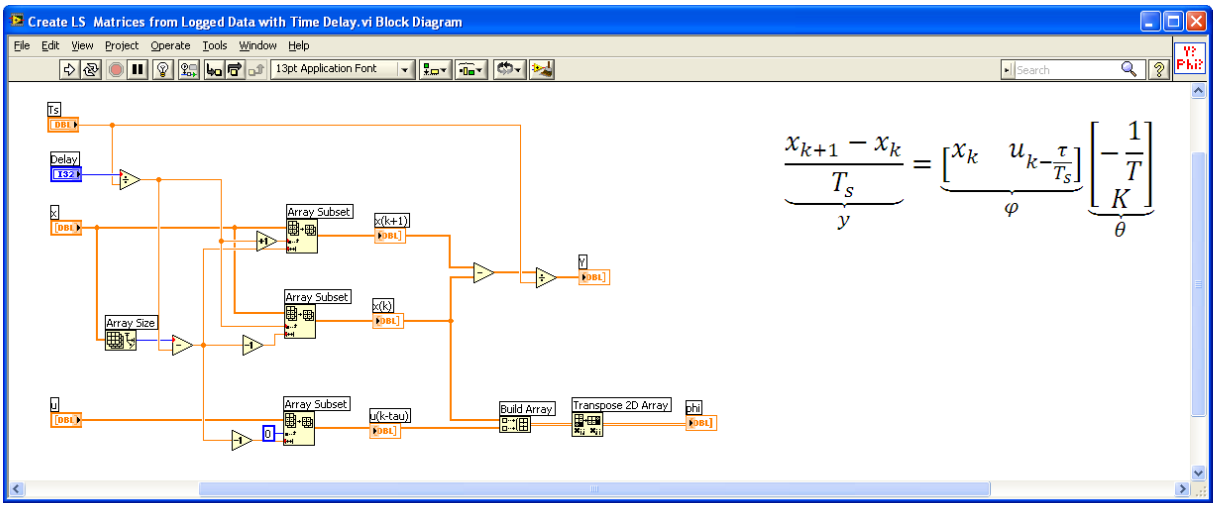


2. "Create LS Matrices from Logged Data with Time Delay.vi"

This SubVI "stack" data on the form:

$$\underbrace{\frac{x_{k+1}-x_k}{T_s}}_{y} = \underbrace{\begin{bmatrix} x_k & u_{k-\frac{\tau}{T_s}} \end{bmatrix}}_{\varphi} \underbrace{\begin{bmatrix} -\frac{1}{T} \\ K \end{bmatrix}}_{\theta}$$

Block Diagram:

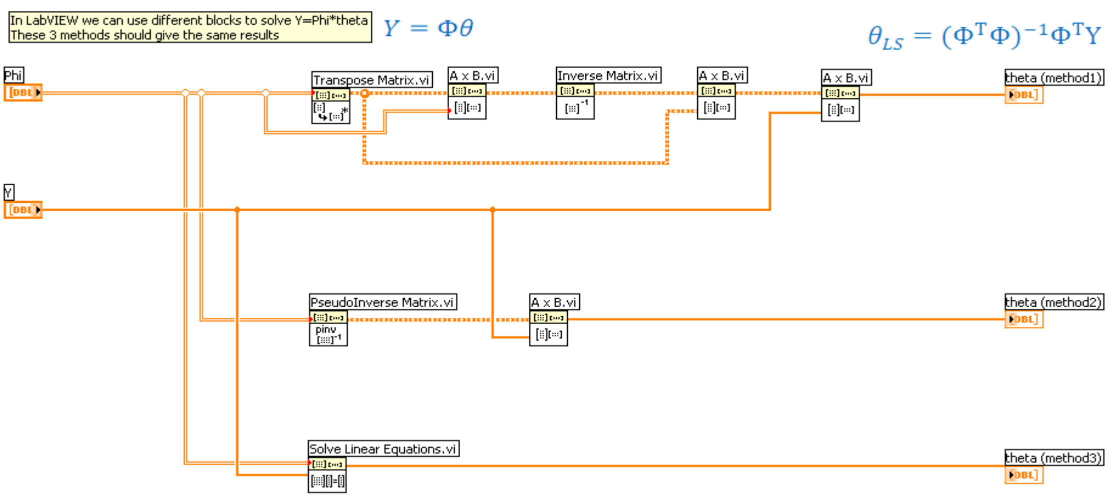


3. "Find LS Solution.vi"

This SubVI find the LS solution:

$$\theta_{LS} = (\Phi^T\Phi)^{-1}\Phi^T Y$$

Block Diagram:



System Identification and Estimation in LabVIEW

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